28 SPECIAL RELATIVITY

Figure 28.1 Special relativity explains why traveling to other star systems, such as these in the Orion Nebula, is unreasonable using our current level of technology. (credit: s58y, Flickr)

Learning Objectives

Introduction to Special Relativity

Have you ever looked up at the night sky and dreamed of traveling to other planets in faraway star systems? Would there be other life forms? What would other worlds look like? You might imagine that such an amazing trip would be possible if we could just travel fast enough, but you will read in this chapter why this is not true. In 1905 Albert Einstein developed the theory of special relativity. This theory explains the limit on an object's speed and describes the consequences.

Relativity. The word *relativity* might conjure an image of Einstein, but the idea did not begin with him. People have been exploring relativity for many centuries. Relativity is the study of how different observers measure the same event. Galileo and Newton developed the first correct version of classical relativity. Einstein developed the modern theory of relativity. Modern relativity is divided into two parts. *Special relativity* deals with observers who are moving at constant velocity. *General relativity* deals with observers who are undergoing acceleration. Einstein is famous because his theories of relativity made revolutionary predictions. Most importantly, his theories have been verified to great precision in a vast range of experiments, altering forever our concept of space and time.

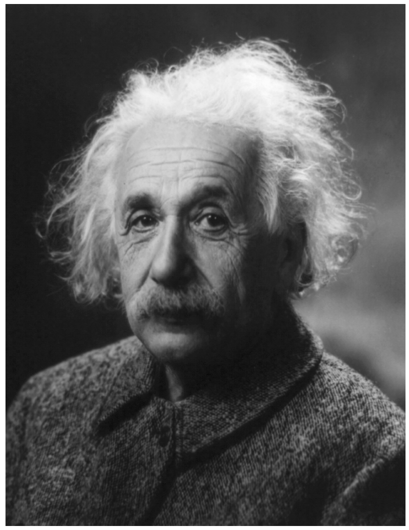


Figure 28.2 Many people think that Albert Einstein (1879–1955) was the greatest physicist of the 20th century. Not only did he develop modern relativity, thus revolutionizing our concept of the universe, he also made fundamental contributions to the foundations of quantum mechanics. (credit: The Library of Congress)

It is important to note that although classical mechanic, in general, and classical relativity, in particular, are limited, they are extremely good approximations for large, slow-moving objects. Otherwise, we could not use classical physics to launch satellites or build bridges. In the classical limit (objects larger than submicroscopic and moving slower than about 1% of the speed of light), relativistic mechanics becomes the same as classical mechanics. This fact will be noted at appropriate places throughout this chapter.

28.1 Einstein's Postulates

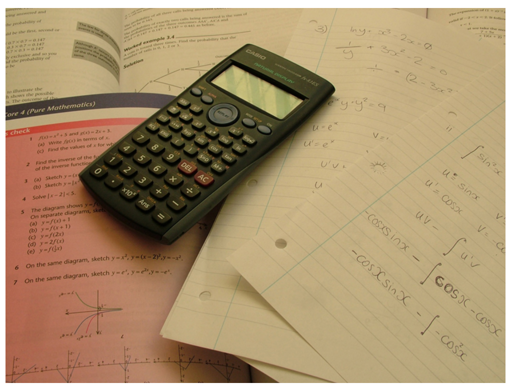


Figure 28.3 Special relativity resembles trigonometry in that both are reliable because they are based on postulates that flow one from another in a logical way. (credit: Jon Oakley, Flickr)

Have you ever used the Pythagorean Theorem and gotten a wrong answer? Probably not, unless you made a mistake in either your algebra or your arithmetic. Each time you perform the same calculation, you know that the answer will be the same. Trigonometry is reliable because of the certainty that one part always flows from another in a logical way. Each part is based on a set of postulates, and you can always connect the parts by applying those postulates. Physics is the same way with the exception that *all* parts must describe nature. If we are careful to choose the correct postulates, then our theory will follow and will be verified by experiment.

Einstein essentially did the theoretical aspect of this method for **relativity**. With two deceptively simple postulates and a careful consideration of how measurements are made, he produced the theory of **special relativity.**

Einstein's First Postulate

The first postulate upon which Einstein based the theory of special relativity relates to reference frames. All velocities are measured relative to some frame of reference. For example, a car's motion is measured relative to its starting point or the road it is moving over, a projectile's motion is measured relative to the surface it was launched from, and a planet's orbit is measured relative to the star it is orbiting around. The simplest frames of reference are those that are not accelerated and are not rotating. Newton's first law, the law of inertia, holds exactly in such a frame.

Inertial Reference Frame

An **inertial frame of reference** is a reference frame in which a body at rest remains at rest and a body in motion moves at a constant speed in a straight line unless acted on by an outside force.

The laws of physics seem to be simplest in inertial frames. For example, when you are in a plane flying at a constant altitude and speed, physics seems to work exactly the same as if you were standing on the surface of the Earth. However, in a plane that is taking off, matters are somewhat more complicated. In these cases, the net force on an object, F , is not equal to the product of mass and acceleration, ma . Instead, F is equal to ma plus a fictitious force. This situation is not as simple as in an inertial frame. Not only are laws of physics simplest in inertial frames, but they should be the same in all inertial frames, since there is no preferred frame and no absolute motion. Einstein incorporated these ideas into his **first postulate of special relativity**.

First Postulate of Special Relativity

The laws of physics are the same and can be stated in their simplest form in all inertial frames of reference.

As with many fundamental statements, there is more to this postulate than meets the eye. The laws of physics include only those that satisfy this postulate. We shall find that the definitions of relativistic momentum and energy must be altered to fit. Another outcome of this postulate is the famous equation $E = mc^2$.

Einstein's Second Postulate

The second postulate upon which Einstein based his theory of special relativity deals with the speed of light. Late in the 19th century, the major tenets of classical physics were well established. Two of the most important were the laws of electricity and magnetism and Newton's laws. In particular, the laws of electricity and magnetism predict that light travels at $c = 3.00\times10^8\text{ m/s}$ in a vacuum, but they do not specify the frame of reference in which light has this speed.

There was a contradiction between this prediction and Newton's laws, in which velocities add like simple vectors. If the latter were true, then two observers moving at different speeds would see light traveling at different speeds. Imagine what a light wave would look like to a person traveling along with it at a speed c. If such a motion were possible then the wave would be stationary relative to the observer. It would have electric and magnetic fields that varied in strength at various distances from the observer but were constant in time. This is not allowed by Maxwell's equations. So either Maxwell's equations are wrong, or an object with mass cannot travel at speed c. Einstein concluded that the latter is true. An object with mass cannot travel at speed c. This conclusion implies that light in a vacuum must always travel at speed c relative to any observer. Maxwell's equations are correct, and Newton's addition of velocities is not correct for light.

Investigations such as Young's double slit experiment in the early-1800s had convincingly demonstrated that light is a wave. Many types of waves were known, and all travelled in some medium. Scientists therefore assumed that a medium carried light, even in a vacuum, and light travelled at a speed c relative to that medium. Starting in the mid-1880s, the American physicist A. A. Michelson, later aided by E. W. Morley, made a series of direct measurements of the speed of light. The results of their measurements were startling.

Michelson-Morley Experiment

The **Michelson-Morley experiment** demonstrated that the speed of light in a vacuum is independent of the motion of the Earth about the Sun.

The eventual conclusion derived from this result is that light, unlike mechanical waves such as sound, does not need a medium to carry it. Furthermore, the Michelson-Morley results implied that the speed of light c is independent of the motion of the source relative to the observer. That is, everyone observes light to move at speed c regardless of how they move relative to the source or one another. For a number of years, many scientists tried unsuccessfully to explain these results and still retain the general applicability of Newton's laws.

It was not until 1905, when Einstein published his first paper on special relativity, that the currently accepted conclusion was reached. Based mostly on his analysis that the laws of electricity and magnetism would not allow another speed for light, and only slightly aware of the Michelson-Morley experiment, Einstein detailed his **second postulate of special relativity**.

Second Postulate of Special Relativity

The speed of light c is a constant, independent of the relative motion of the source.

Deceptively simple and counterintuitive, this and the first postulate leave all else open for change. Some fundamental concepts do change. Among the changes are the loss of agreement on the elapsed time for an event, the variation of distance with speed, and the realization that matter and energy can be converted into one another. You will read about these concepts in the following sections.

Misconception Alert: Constancy of the Speed of Light

The speed of light is a constant $c = 3.00\times10^8\text{ m/s}$ *in a vacuum*. If you remember the effect of the index of refraction from **The Law of Refraction**, the speed of light is lower in matter.

Check Your Understanding

Explain how special relativity differs from general relativity.

Solution

Special relativity applies only to unaccelerated motion, but general relativity applies to accelerated motion.

28.2 Simultaneity And Time Dilation

Figure 28.4 Elapsed time for a foot race is the same for all observers, but at relativistic speeds, elapsed time depends on the relative motion of the observer and the event that is observed. (credit: Jason Edward Scott Bain, Flickr)

Do time intervals depend on who observes them? Intuitively, we expect the time for a process, such as the elapsed time for a foot race, to be the same for all observers. Our experience has been that disagreements over elapsed time have to do with the accuracy of measuring time. When we carefully consider just how time is measured, however, we will find that elapsed time depends on the relative motion of an observer with respect to the process being measured.

Simultaneity

Consider how we measure elapsed time. If we use a stopwatch, for example, how do we know when to start and stop the watch? One method is to use the arrival of light from the event, such as observing a light turning green to start a drag race. The timing will be more accurate if some sort of electronic detection is used, avoiding human reaction times and other complications.

Now suppose we use this method to measure the time interval between two flashes of light produced by flash lamps. (See **Figure 28.5**.) Two flash lamps with observer A midway between them are on a rail car that moves to the right relative to observer B. The light flashes are emitted just as A passes B, so that both A and B are equidistant from the lamps when the light is emitted. Observer B measures the time interval between the arrival of the light flashes. According to postulate 2, the speed of light is not affected by the motion of the lamps relative to B. Therefore, light travels equal distances to him at equal speeds. Thus observer B measures the flashes to be simultaneous.

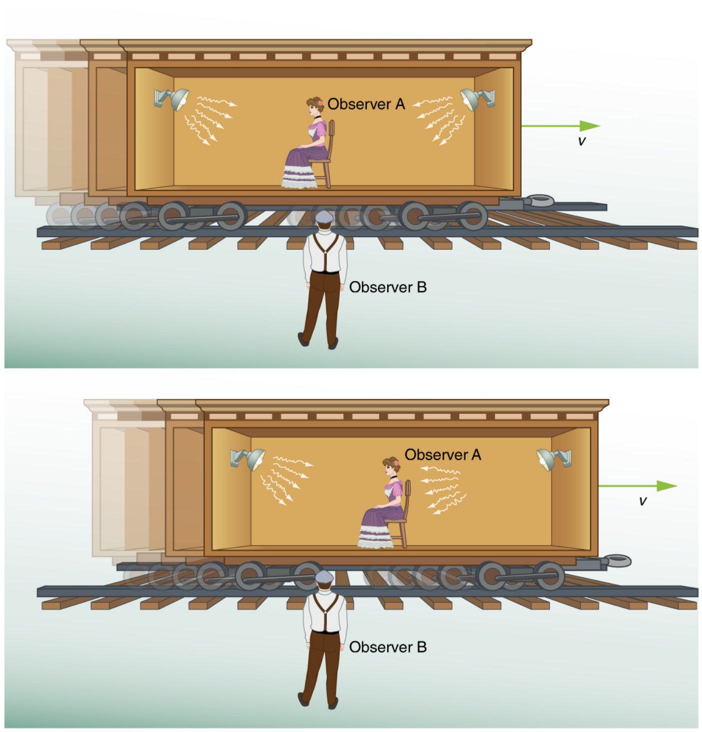


Figure 28.5 Observer B measures the elapsed time between the arrival of light flashes as described in the text. Observer A moves with the lamps on a rail car. Observer B receives the light flashes simultaneously, but he notes that observer A receives the flash from the right first. B observes the flashes to be simultaneous to him but not to A. Simultaneity is not absolute.

Now consider what observer B sees happen to observer A. She receives the light from the right first, because she has moved towards that flash lamp, lessening the distance the light must travel and reducing the time it takes to get to her. Light travels at speed c relative to both observers, but observer B remains equidistant between the points where the flashes were emitted, while A gets closer to the emission point on the right. From observer B’s point of view, then, there is a time interval between the arrival of the flashes to observer A. Observer B measures the flashes to be simultaneous relative to him but not relative to A. Here a relative velocity between observers affects whether two events are observed to be simultaneous. *Simultaneity is not absolute*.

This illustrates the power of clear thinking. We might have guessed incorrectly that if light is emitted simultaneously, then two observers halfway between the sources would see the flashes simultaneously. But careful analysis shows this not to be the case. Einstein was brilliant at this type of *thought experiment* (in German, “Gedankenexperiment”). He very carefully considered how an observation is made and disregarded what might seem obvious. The validity of thought experiments, of course, is determined by actual observation. The genius of Einstein is evidenced by the fact that experiments have repeatedly confirmed his theory of relativity.

In summary: Two events are defined to be simultaneous if an observer measures them as occurring at the same time (such as by receiving light from the events). Two events are not necessarily simultaneous to all observers.

Time Dilation

The consideration of the measurement of elapsed time and simultaneity leads to an important relativistic effect.

Time dilation

Time dilation is the phenomenon of time passing slower for an observer who is moving relative to another observer.

Suppose, for example, an astronaut measures the time it takes for light to cross her ship, bounce off a mirror, and return. (See **Figure 28.6**.) How does the elapsed time the astronaut measures compare with the elapsed time measured for the same event by a person on the Earth? Asking this question (another thought experiment) produces a profound result. We find that the elapsed time for a process depends on who is measuring it. In this case, the time measured by the astronaut is smaller than the time measured by the Earth-bound observer. The passage of time is different for the observers because the distance the light travels in the astronaut’s frame is smaller than in the Earth-bound frame. Light travels at the same speed in each frame, and so it will take longer to travel the greater distance in the Earth-bound frame.

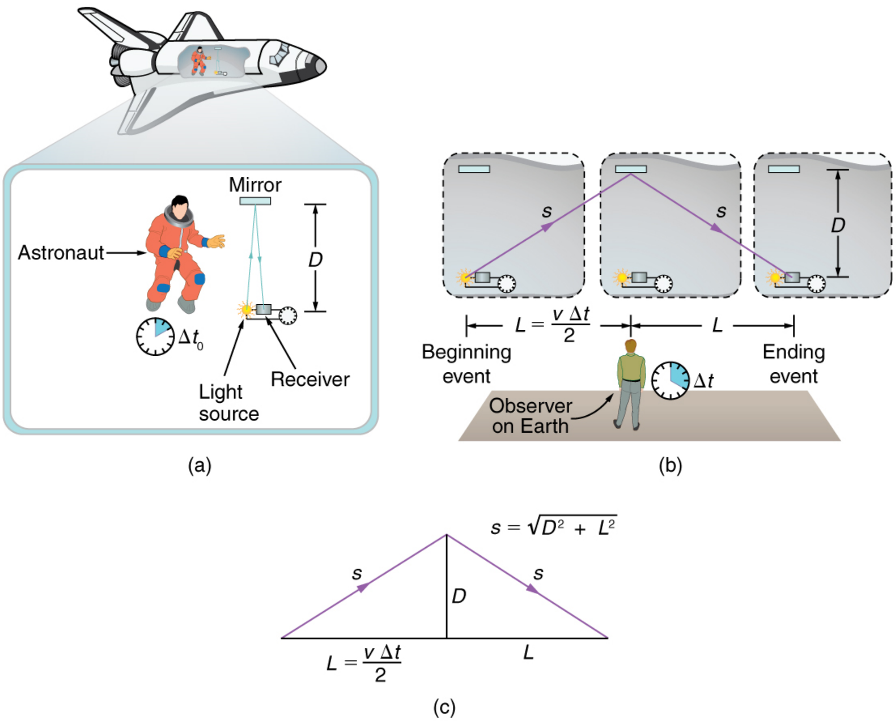


Figure 28.6 (a) An astronaut measures the time Δt_0 for light to cross her ship using an electronic timer. Light travels a distance $2D$ in the astronaut’s frame. (b) A person on the Earth sees the light follow the longer path $2s$ and take a longer time Δt . (c) These triangles are used to find the relationship between the two distances $2D$ and $2s$.

To quantitatively verify that time depends on the observer, consider the paths followed by light as seen by each observer. (See **Figure 28.6**(c).) The astronaut sees the light travel straight across and back for a total distance of $2D$, twice the width of her ship. The Earth-bound observer sees the light travel a total distance $2s$. Since the ship is moving at speed v to the right relative to the Earth, light moving to the right hits the mirror in this frame. Light travels at a speed c in both frames, and because time is the distance divided by speed, the time measured by the astronaut is

$$\Delta t_0 = \frac{2D}{c}. \tag{28.1}$$

This time has a separate name to distinguish it from the time measured by the Earth-bound observer.

Proper Time

Proper time Δt_0 is the time measured by an observer at rest relative to the event being observed.

In the case of the astronaut observe the reflecting light, the astronaut measures proper time. The time measured by the Earth-bound observer is

$$\Delta t = \frac{2s}{c}. \tag{28.2}$$

To find the relationship between Δt_0 and Δt , consider the triangles formed by D and s . (See **Figure 28.6**(c).) The third side of these similar triangles is L , the distance the astronaut moves as the light goes across her ship. In the frame of the Earth-bound observer,

$$L = \frac{v\Delta t}{2}. \tag{28.3}$$

Using the Pythagorean Theorem, the distance s is found to be

$$s = \sqrt{D^2 + \left(\frac{v\Delta t}{2}\right)^2}. \tag{28.4}$$

Substituting s into the expression for the time interval Δt gives

$$\Delta t = \frac{2s}{c} = \frac{2\sqrt{D^2 + \left(\frac{v\Delta t}{2}\right)^2}}{c}. \tag{28.5}$$

We square this equation, which yields

$$(\Delta t)^2 = \frac{4\left(D^2 + \frac{v^2(\Delta t)^2}{4}\right)}{c^2} = \frac{4D^2}{c^2} + \frac{v^2}{c^2}(\Delta t)^2. \tag{28.6}$$

Note that if we square the first expression we had for Δt_0 , we get $(\Delta t_0)^2 = \frac{4D^2}{c^2}$. This term appears in the preceding equation, giving us a means to relate the two time intervals. Thus,

$$(\Delta t)^2 = (\Delta t_0)^2 + \frac{v^2}{c^2}(\Delta t)^2. \tag{28.7}$$

Gathering terms, we solve for Δt :

$$(\Delta t)^2\left(1 - \frac{v^2}{c^2}\right) = (\Delta t_0)^2. \tag{28.8}$$

Thus,

$$(\Delta t)^2 = \frac{(\Delta t_0)^2}{1 - \frac{v^2}{c^2}}. \tag{28.9}$$

Taking the square root yields an important relationship between elapsed times:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \Delta t_0, \tag{28.10}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{28.11}$$

This equation for Δt is truly remarkable. First, as contended, elapsed time is not the same for different observers moving relative to one another, even though both are in inertial frames. Proper time Δt_0 measured by an observer, like the astronaut moving with the apparatus, is smaller than time measured by other observers. Since those other observers measure a longer time Δt , the effect is called time dilation. The Earth-bound observer sees time dilate (get longer) for a system moving relative to the Earth. Alternatively, according to the Earth-bound observer, time slows in the moving frame, since less time passes there. All clocks moving relative to an observer, including biological clocks such as aging, are observed to run slow compared with a clock stationary relative to the observer.

Note that if the relative velocity is much less than the speed of light ($v << c$), then $\frac{v^2}{c^2}$ is extremely small, and the elapsed times Δt and Δt_0 are nearly equal. At low velocities, modern relativity approaches classical physics—our everyday experiences have very small relativistic effects.

The equation $\Delta t = \gamma \Delta t_0$ also implies that relative velocity cannot exceed the speed of light. As v approaches c , Δt approaches infinity. This would imply that time in the astronaut's frame stops at the speed of light. If v exceeded c , then we would be taking the square root of a negative number, producing an imaginary value for Δt .

There is considerable experimental evidence that the equation $\Delta t = \gamma \Delta t_0$ is correct. One example is found in cosmic ray particles that continuously rain down on the Earth from deep space. Some collisions of these particles with nuclei in the upper atmosphere result in short-lived particles called muons. The half-life (amount of time for half of a material to decay) of a muon is $1.52\ \mu s$ when it is at rest relative to the observer who measures the half-life. This is the proper time Δt_0 . Muons produced by cosmic ray particles have a range of velocities, with some moving near the speed of light. It has been found that the muon's half-life as measured by an Earth-bound observer (Δt) varies with velocity exactly as predicted by the equation $\Delta t = \gamma \Delta t_0$. The faster the muon moves, the longer it lives. We on the Earth see the muon's half-life time dilated—as viewed from our frame, the muon decays more slowly than it does when at rest relative to us.

Example 28.1 Calculating Δt for a Relativistic Event: How Long Does a Speedy Muon Live?

Suppose a cosmic ray colliding with a nucleus in the Earth's upper atmosphere produces a muon that has a velocity $v = 0.950c$. The muon then travels at constant velocity and lives $1.52\ \mu s$ as measured in the muon's frame of reference. (You can imagine this as the muon's internal clock.) How long does the muon live as measured by an Earth-bound observer? (See **Figure 28.7**.)

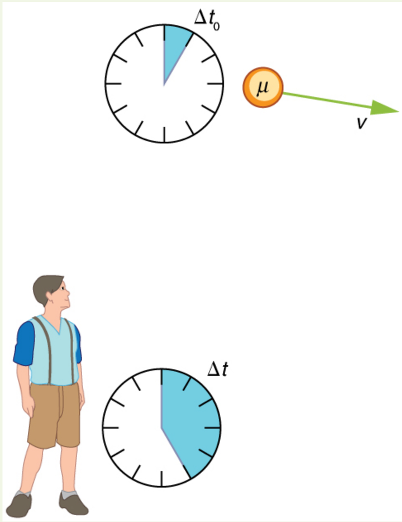


Figure 28.7 A muon in the Earth's atmosphere lives longer as measured by an Earth-bound observer than measured by the muon's internal clock.

Strategy

A clock moving with the system being measured observes the proper time, so the time we are given is $\Delta t_0 = 1.52\ \mu s$. The Earth-bound observer measures Δt as given by the equation $\Delta t = \gamma \Delta t_0$. Since we know the velocity, the calculation is straightforward.

Solution

1) Identify the knowns. $v = 0.950c$, $\Delta t_0 = 1.52\ \mu s$

2) Identify the unknown. Δt

3) Choose the appropriate equation.

Use,

$$\Delta t = \gamma \Delta t_0, \tag{28.12}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{28.13}$$

4) Plug the knowns into the equation.

First find γ .

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \\ &= \frac{1}{\sqrt{1 - \frac{(0.950c)^2}{c^2}}} \\ &= \frac{1}{\sqrt{1 - (0.950)^2}} \\ &= 3.20. \end{aligned} \tag{28.14}$$

Use the calculated value of γ to determine Δt .

$$\begin{aligned} \Delta t &= \gamma \Delta t_0 \\ &= (3.20)(1.52\ \mu s) \\ &= 4.87\ \mu s \end{aligned} \tag{28.15}$$

Discussion

One implication of this example is that since $\gamma = 3.20$ at 95.0% of the speed of light ($v = 0.950c$), the relativistic effects are significant. The two time intervals differ by this factor of 3.20, where classically they would be the same. Something moving at $0.950c$ is said to be highly relativistic.

Another implication of the preceding example is that everything an astronaut does when moving at 95.0% of the speed of light relative to the Earth takes 3.20 times longer when observed from the Earth. Does the astronaut sense this? Only if she looks outside her spaceship. All methods of measuring time in her frame will be affected by the same factor of 3.20. This includes her wristwatch, heart rate, cell metabolism rate, nerve impulse rate, and so on. She will have no way of telling, since all of her clocks will agree with one another because their relative velocities are zero. Motion is relative, not absolute. But what if she does look out the window?

Real-World Connections

It may seem that special relativity has little effect on your life, but it is probably more important than you realize. One of the most common effects is through the Global Positioning System (GPS). Emergency vehicles, package delivery services, electronic maps, and communications devices are just a few of the common uses of GPS, and the GPS system could not work without taking into account relativistic effects. GPS satellites rely on precise time measurements to communicate. The signals travel at relativistic speeds. Without corrections for time dilation, the satellites could not communicate, and the GPS system would fail within minutes.

The Twin Paradox

An intriguing consequence of time dilation is that a space traveler moving at a high velocity relative to the Earth would age less than her Earth-bound twin. Imagine the astronaut moving at such a velocity that $\gamma = 30.0$, as in **Figure 28.8**. A trip that takes 2.00 years in her frame would take 60.0 years in her Earth-bound twin's frame. Suppose the astronaut traveled 1.00 year to another star system. She briefly explored the area, and then traveled 1.00 year back. If the astronaut was 40 years old when she left, she would be 42 upon her return. Everything on the Earth, however, would have aged 60.0 years. Her twin, if still alive, would be 100 years old.

The situation would seem different to the astronaut. Because motion is relative, the spaceship would seem to be stationary and the Earth would appear to move. (This is the sensation you have when flying in a jet.) If the astronaut looks out the window of the spaceship, she will see time slow down on the Earth by a factor of $\gamma = 30.0$. To her, the Earth-bound sister will have aged only 2/30 (1/15) of a year, while she aged 2.00 years. The two sisters cannot both be correct.

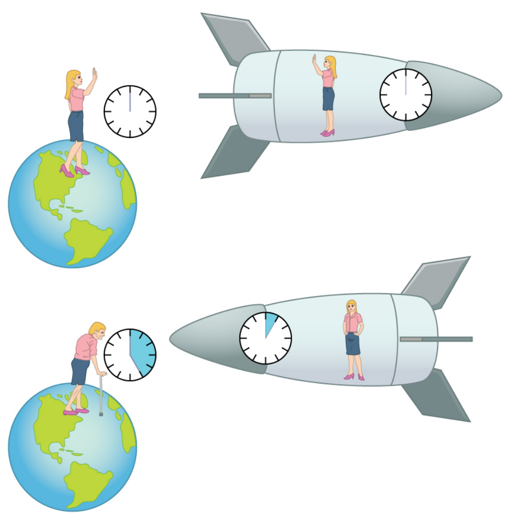


Figure 28.8 The twin paradox asks why the traveling twin ages less than the Earth-bound twin. That is the prediction we obtain if we consider the Earth-bound twin's frame. In the astronaut's frame, however, the Earth is moving and time runs slower there. Who is correct?

As with all paradoxes, the premise is faulty and leads to contradictory conclusions. In fact, the astronaut's motion is significantly different from that of the Earth-bound twin. The astronaut accelerates to a high velocity and then decelerates to view the star system. To return to the Earth, she again accelerates and decelerates. The Earth-bound twin does not experience these accelerations. So the situation is not symmetric, and it is not correct to claim that the astronaut will observe the same effects as her Earth-bound twin. If you use special relativity to examine the twin paradox, you must keep in mind that the theory is expressly based on inertial frames, which by definition are not accelerated or rotating. Einstein developed general relativity to deal with accelerated frames and with gravity, a prime source of acceleration. You can also use general relativity to address the twin paradox and, according to general relativity, the astronaut will age less. Some important conceptual aspects of general relativity are discussed in **General Relativity and Quantum Gravity** of this course.

In 1971, American physicists Joseph Hafele and Richard Keating verified time dilation at low relative velocities by flying extremely accurate atomic clocks around the Earth on commercial aircraft. They measured elapsed time to an accuracy of a few nanoseconds and compared it with the time measured by clocks left behind. Hafele and Keating's results were within experimental uncertainties of the predictions of relativity. Both special and general relativity had to be taken into account, since gravity and accelerations were involved as well as relative motion.

Check Your Understanding

1. What is γ if $v = 0.650c$?

Solution

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.650c)^2}{c^2}}} = 1.32$$

2. A particle travels at $1.90 \times 10^8 \text{ m/s}$ and lives $2.10 \times 10^{-8} \text{ s}$ when at rest relative to an observer. How long does the particle live as viewed in the laboratory?

Solution

$$\Delta t = \frac{\Delta_t}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2.10 \times 10^{-8} \text{ s}}{\sqrt{1 - \frac{(1.90 \times 10^8 \text{ m/s})^2}{(3.00 \times 10^8 \text{ m/s})^2}}} = 2.71 \times 10^{-8} \text{ s}$$

28.3 Length Contraction

Figure 28.9 People might describe distances differently, but at relativistic speeds, the distances really are different. (credit: Corey Leopold, Flickr)

Have you ever driven on a road that seems like it goes on forever? If you look ahead, you might say you have about 10 km left to go. Another traveler might say the road ahead looks like it's about 15 km long. If you both measured the road, however, you would agree. Traveling at everyday speeds, the distance you both measure would be the same. You will read in this section, however, that this is not true at relativistic speeds. Close to the speed of light, distances measured are not the same when measured by different observers.

Proper Length

One thing all observers agree upon is relative speed. Even though clocks measure different elapsed times for the same process, they still agree that relative speed, which is distance divided by elapsed time, is the same. This implies that distance, too, depends on the observer's relative motion. If two observers see different times, then they must also see different distances for relative speed to be the same to each of them.

The muon discussed in **Example 28.1** illustrates this concept. To an observer on the Earth, the muon travels at $0.950c$ for $7.05 \ \mu\text{s}$ from the time it is produced until it decays. Thus it travels a distance

$$L_0 = v\Delta t = (0.950)(3.00 \times 10^8 \text{ m/s})(7.05 \times 10^{-6} \text{ s}) = 2.01 \text{ km} \tag{28.16}$$

relative to the Earth. In the muon's frame of reference, its lifetime is only $2.20 \ \mu\text{s}$. It has enough time to travel only

$$L = v\Delta t_0 = (0.950)(3.00 \times 10^8 \text{ m/s})(2.20 \times 10^{-6} \text{ s}) = 0.627 \text{ km}. \tag{28.17}$$

The distance between the same two events (production and decay of a muon) depends on who measures it and how they are moving relative to it.

Proper Length

Proper length L_0 is the distance between two points measured by an observer who is at rest relative to both of the points.

The Earth-bound observer measures the proper length L_0 , because the points at which the muon is produced and decays are stationary relative to the Earth. To the muon, the Earth, air, and clouds are moving, and so the distance L it sees is not the proper length.

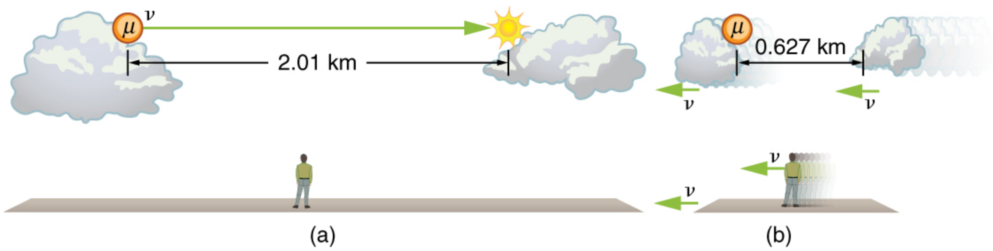


Figure 28.10 (a) The Earth-bound observer sees the muon travel 2.01 km between clouds. (b) The muon sees itself travel the same path, but only a distance of 0.627 km. The Earth, air, and clouds are moving relative to the muon in its frame, and all appear to have smaller lengths along the direction of travel.

Length Contraction

To develop an equation relating distances measured by different observers, we note that the velocity relative to the Earth-bound observer in our muon example is given by

$$v = \frac{L_0}{\Delta t}. \tag{28.18}$$

The time relative to the Earth-bound observer is Δt , since the object being timed is moving relative to this observer. The velocity relative to the moving observer is given by

$$v = \frac{L}{\Delta t_0}. \tag{28.19}$$

The moving observer travels with the muon and therefore observes the proper time Δt_0 . The two velocities are identical; thus,

$$\frac{L_0}{\Delta t} = \frac{L}{\Delta t_0}. \tag{28.20}$$

We know that $\Delta t = \gamma \Delta t_0$. Substituting this equation into the relationship above gives

$$L = \frac{L_0}{\gamma}. \tag{28.21}$$

Substituting for γ gives an equation relating the distances measured by different observers.

Length Contraction

Length contraction L is the shortening of the measured length of an object moving relative to the observer's frame.

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}. \tag{28.22}$$

If we measure the length of anything moving relative to our frame, we find its length L to be smaller than the proper length L_0 that would be measured if the object were stationary. For example, in the muon's reference frame, the distance between the points where it was produced and where it decayed is shorter. Those points are fixed relative to the Earth but moving relative to the muon. Clouds and other objects are also contracted along the direction of motion in the muon's reference frame.

Example 28.2 Calculating Length Contraction: The Distance between Stars Contracts when You Travel at High Velocity

Suppose an astronaut, such as the twin discussed in **Simultaneity and Time Dilation**, travels so fast that $\gamma = 30.00$. (a) She travels from the Earth to the nearest star system, Alpha Centauri, 4.300 light years (ly) away as measured by an Earth-bound observer. How far apart are the Earth and Alpha Centauri as measured by the astronaut? (b) In terms of c , what is her velocity relative to the Earth? You may neglect the motion of the Earth relative to the Sun. (See **Figure 28.11**.)

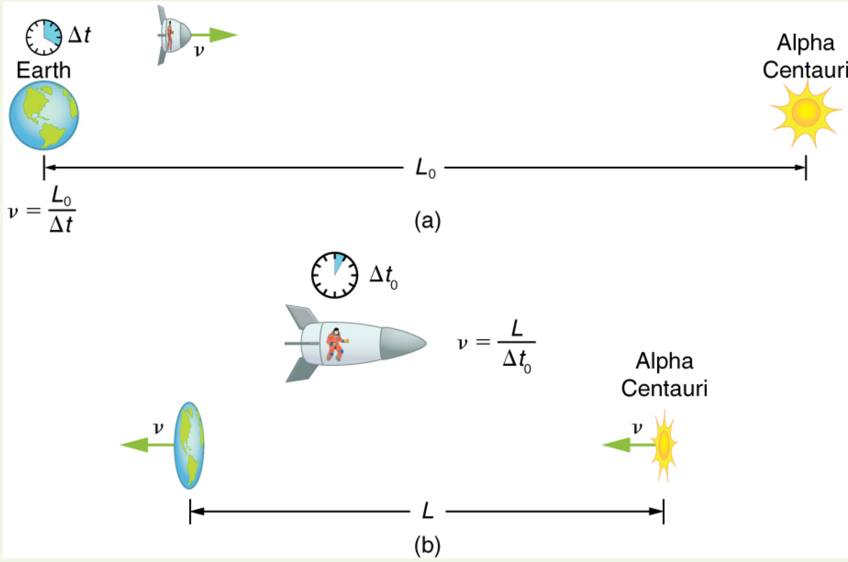


Figure 28.11 (a) The Earth-bound observer measures the proper distance between the Earth and the Alpha Centauri. (b) The astronaut observes a length contraction, since the Earth and the Alpha Centauri move relative to her ship. She can travel this shorter distance in a smaller time (her proper time) without exceeding the speed of light.

Strategy

First note that a light year (ly) is a convenient unit of distance on an astronomical scale—it is the distance light travels in a year. For part (a), note that the 4.300 ly distance between the Alpha Centauri and the Earth is the proper distance L_0 , because it is measured by an Earth-bound observer to whom both stars are (approximately) stationary. To the astronaut, the Earth and the Alpha Centauri are moving by at the same velocity, and so the distance between them is the contracted length L . In part (b), we are given γ , and so we can find v by rearranging the definition of γ to express v in terms of c .

Solution for (a)

1. Identify the knowns. $L_0 - 4.300 \text{ ly}$; $\gamma = 30.00$
2. Identify the unknown. L
3. Choose the appropriate equation. $L = \frac{L_0}{\gamma}$
4. Rearrange the equation to solve for the unknown.

$$\begin{aligned} L &= \frac{L_0}{\gamma} \\ &= \frac{4.300 \text{ ly}}{30.00} \\ &= 0.1433 \text{ ly} \end{aligned} \tag{28.23}$$

Solution for (b)

1. Identify the known. $\gamma = 30.00$
2. Identify the unknown. v in terms of c
3. Choose the appropriate equation. $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
4. Rearrange the equation to solve for the unknown.

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \\ 30.00 &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned} \tag{28.24}$$

Squaring both sides of the equation and rearranging terms gives

$$900.0 = \frac{1}{1 - \frac{v^2}{c^2}} \tag{28.25}$$

so that

$$1 - \frac{v^2}{c^2} = \frac{1}{900.0} \tag{28.26}$$

and

$$\frac{v^2}{c^2} = 1 - \frac{1}{900.0} = 0.99888.... \tag{28.27}$$

Taking the square root, we find

$$\frac{v}{c} = 0.99944, \tag{28.28}$$

which is rearranged to produce a value for the velocity

$$v = 0.9994c. \tag{28.29}$$

Discussion

First, remember that you should not round off calculations until the final result is obtained, or you could get erroneous results. This is especially true for special relativity calculations, where the differences might only be revealed after several decimal places. The relativistic effect is large here ($\gamma=30.00$), and we see that v is approaching (not equaling) the speed of light. Since the distance as measured by the astronaut is so much smaller, the astronaut can travel it in much less time in her frame.

People could be sent very large distances (thousands or even millions of light years) and age only a few years on the way if they traveled at extremely high velocities. But, like emigrants of centuries past, they would leave the Earth they know forever. Even if they returned, thousands to millions of years would have passed on the Earth, obliterating most of what now exists. There is also a more serious practical obstacle to traveling at such velocities; immensely greater energies than classical physics predicts would be needed to achieve such high velocities. This will be discussed in **Relatavistic Energy**.

Why don't we notice length contraction in everyday life? The distance to the grocery shop does not seem to depend on whether we are moving or not. Examining the equation $L = L_0\sqrt{1 - \frac{v^2}{c^2}}$, we see that at low velocities ($v<<c$) the lengths are nearly equal, the classical expectation. But length contraction is real, if not commonly experienced. For example, a charged particle, like an electron, traveling at relativistic velocity has electric field lines that are compressed along the direction of motion as seen by a stationary observer. (See **Figure 28.12**.) As the electron passes a detector, such as a coil of wire, its field interacts much more briefly, an effect observed at particle accelerators such as the 3 km long Stanford Linear Accelerator (SLAC). In fact, to an electron traveling down the beam pipe at SLAC, the accelerator and the Earth are all moving by and are length contracted. The relativistic effect is so great than the accelerator is only 0.5 m long to the electron. It is actually easier to get the electron beam down the pipe, since the beam does not have to be as precisely aimed to get down a short pipe as it would down one 3 km long. This, again, is an experimental verification of the Special Theory of Relativity.

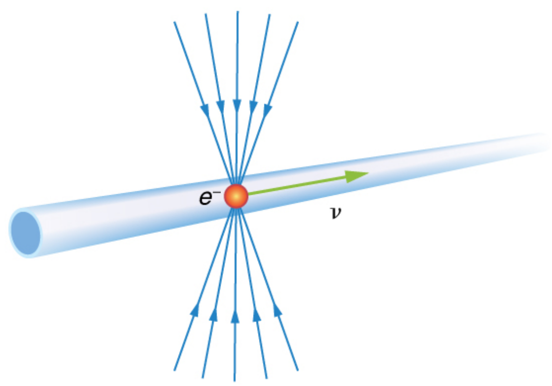


Figure 28.12 The electric field lines of a high-velocity charged particle are compressed along the direction of motion by length contraction. This produces a different signal when the particle goes through a coil, an experimentally verified effect of length contraction.

Check Your Understanding

A particle is traveling through the Earth's atmosphere at a speed of $0.750c$. To an Earth-bound observer, the distance it travels is 2.50 km. How far does the particle travel in the particle's frame of reference?

Solution

$$L=L_0\sqrt{1 - \frac{v^2}{c^2}} = (2.50\text{ km})\sqrt{1 - \frac{(0.750c)^2}{c^2}} = 1.65\text{ km} \tag{28.30}$$

28.4 Relativistic Addition of Velocities

Figure 28.13 The total velocity of a kayak, like this one on the Deerfield River in Massachusetts, is its velocity relative to the water as well as the water's velocity relative to the riverbank. (credit: abkfenris, Flickr)

If you've ever seen a kayak move down a fast-moving river, you know that remaining in the same place would be hard. The river current pulls the kayak along. Pushing the oars back against the water can move the kayak forward in the water, but that only accounts for part of the velocity. The kayak's motion is an example of classical addition of velocities. In classical physics, velocities add as vectors. The kayak's velocity is the vector sum of its velocity relative to the water and the water's velocity relative to the riverbank.

Classical Velocity Addition

For simplicity, we restrict our consideration of velocity addition to one-dimensional motion. Classically, velocities add like regular numbers in one-dimensional motion. (See **Figure 28.14**.) Suppose, for example, a girl is riding in a sled at a speed 1.0 m/s relative to an observer. She throws a snowball first forward, then backward at a speed of 1.5 m/s relative to the sled. We denote direction with plus and minus signs in one dimension; in this example, forward is positive. Let v be the velocity of the sled relative to the Earth, u the velocity of the snowball relative to the Earth-bound observer, and u' the velocity of the snowball relative to the sled.

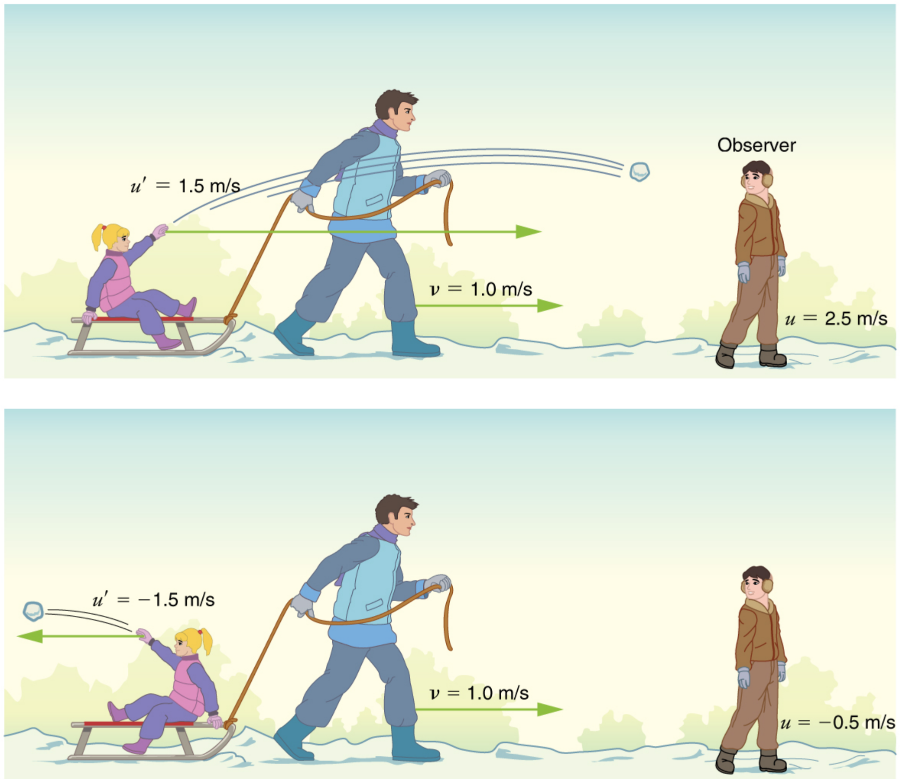


Figure 28.14 Classically, velocities add like ordinary numbers in one-dimensional motion. Here the girl throws a snowball forward and then backward from a sled. The velocity of the sled relative to the Earth is $v = 1.0\ \text{m/s}$. The velocity of the snowball relative to the truck is u', while its velocity relative to the Earth is u. Classically, $u = v + u'$.

Classical Velocity Addition

$$u = v + u' \tag{28.31}$$

Thus, when the girl throws the snowball forward, $u = 1.0\ \text{m/s} + 1.5\ \text{m/s} = 2.5\ \text{m/s}$. It makes good intuitive sense that the snowball will head towards the Earth-bound observer faster, because it is thrown forward from a moving vehicle. When the girl throws the snowball backward, $u = 1.0\ \text{m/s} + (-1.5\ \text{m/s}) = -0.5\ \text{m/s}$. The minus sign means the snowball moves away from the Earth-bound observer.

Relativistic Velocity Addition

The second postulate of relativity (verified by extensive experimental observation) says that classical velocity addition does not apply to light. Imagine a car traveling at night along a straight road, as in **Figure 28.15**. If classical velocity addition applied to light, then the light from the car's headlights would approach the observer on the sidewalk at a speed $u=v+c$. But we know that light will move away from the car at speed c relative to the driver of the car, and light will move towards the observer on the sidewalk at speed c , too.

Figure 28.15 According to experiment and the second postulate of relativity, light from the car's headlights moves away from the car at speed c and towards the observer on the sidewalk at speed c . Classical velocity addition is not valid.

Relativistic Velocity Addition

Either light is an exception, or the classical velocity addition formula only works at low velocities. The latter is the case. The correct formula for one-dimensional **relativistic velocity addition** is

$$u = \frac{v+u'}{1+\frac{vu'}{c^2}}, \tag{28.32}$$

where v is the relative velocity between two observers, u is the velocity of an object relative to one observer, and u' is the velocity relative to the other observer. (For ease of visualization, we often choose to measure u in our reference frame, while someone moving at v relative to us measures u' .) Note that the term $\frac{vu'}{c^2}$ becomes very small at low velocities, and $u = \frac{v+u'}{1+\frac{vu'}{c^2}}$ gives a result very close to classical velocity addition. As before, we see that classical velocity addition is an excellent approximation to the correct relativistic formula for small velocities. No wonder that it seems correct in our experience.

Example 28.3 Showing that the Speed of Light towards an Observer is Constant (in a Vacuum): The Speed of Light is the Speed of Light

Suppose a spaceship heading directly towards the Earth at half the speed of light sends a signal to us on a laser-produced beam of light. Given that the light leaves the ship at speed c as observed from the ship, calculate the speed at which it approaches the Earth.

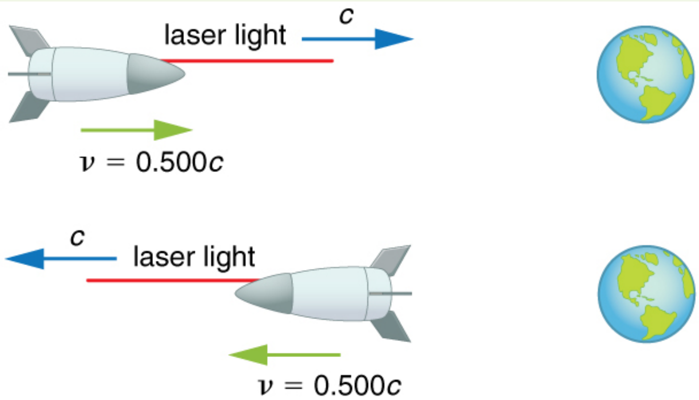


Figure 28.16

Strategy

Because the light and the spaceship are moving at relativistic speeds, we cannot use simple velocity addition. Instead, we can determine the speed at which the light approaches the Earth using relativistic velocity addition.

Solution

1. Identify the knowns. $v=0.500c$; $u' = c$
2. Identify the unknown. u
3. Choose the appropriate equation. $u = \dfrac{v+u'}{1+\frac{vu'}{c^2}}$
4. Plug the knowns into the equation.

$$\begin{aligned} u &= \frac{v+u'}{1+\frac{vu'}{c^2}} \\ &= \frac{0.500c+c}{1+\frac{(0.500c)(c)}{c^2}} \\ &= \frac{(0.500+1)c}{1+\frac{0.500c^2}{c^2}} \\ &= \frac{1.500c}{1+0.500} \\ &= \frac{1.500c}{1.500} \\ &= c \end{aligned} \tag{28.33}$$

Discussion

Relativistic velocity addition gives the correct result. Light leaves the ship at speed c and approaches the Earth at speed c. The speed of light is independent of the relative motion of source and observer, whether the observer is on the ship or Earth-bound.

Velocities cannot add to greater than the speed of light, provided that v is less than c and u' does not exceed c. The following example illustrates that relativistic velocity addition is not as symmetric as classical velocity addition.

Example 28.4 Comparing the Speed of Light towards and away from an Observer: Relativistic Package Delivery

Suppose the spaceship in the previous example is approaching the Earth at half the speed of light and shoots a canister at a speed of $0.750c$. (a) At what velocity will an Earth-bound observer see the canister if it is shot directly towards the Earth? (b) If it is shot directly away from the Earth? (See Figure 28.17.)

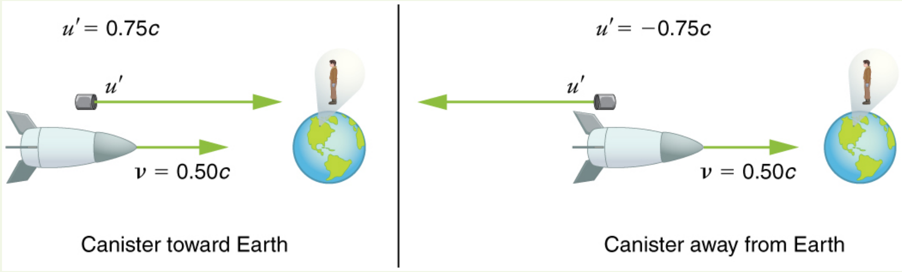


Figure 28.17

Strategy

Because the canister and the spaceship are moving at relativistic speeds, we must determine the speed of the canister by an Earth-bound observer using relativistic velocity addition instead of simple velocity addition.

Solution for (a)

1. Identify the knowns. $v=0.500c$; $u' = 0.750c$
2. Identify the unknown. u
3. Choose the appropriate equation. $u=\dfrac{v+u'}{1+\frac{vu'}{c^2}}$
4. Plug the knowns into the equation.

$$\begin{aligned} u &= \frac{v+u'}{1+\frac{vu'}{c^2}} \\ &= \frac{0.500c + 0.750c}{1+\frac{(0.500c)(0.750c)}{c^2}} \\ &= \frac{1.250c}{1+0.375} \\ &= 0.909c \end{aligned} \tag{28.34}$$

Solution for (b)

1. Identify the knowns. $v = 0.500c$; $u' = -0.750c$
2. Identify the unknown. u
3. Choose the appropriate equation. $u = \frac{v+u'}{1+\frac{vu'}{c^2}}$
4. Plug the knowns into the equation.

$$\begin{aligned} u &= \frac{v+u'}{1+\frac{vu'}{c^2}} \\ &= \frac{0.500c + (-0.750c)}{1+\frac{(0.500c)(-0.750c)}{c^2}} \\ &= \frac{-0.250c}{1-0.375} \\ &= -0.400c \end{aligned} \tag{28.35}$$

Discussion

The minus sign indicates velocity away from the Earth (in the opposite direction from v), which means the canister is heading towards the Earth in part (a) and away in part (b), as expected. But relativistic velocities do not add as simply as they do classically. In part (a), the canister does approach the Earth faster, but not at the simple sum of $1.250c$. The total velocity is less than you would get classically. And in part (b), the canister moves away from the Earth at a velocity of $-0.400c$, which is *faster* than the $-0.250c$ you would expect classically. The velocities are not even symmetric. In part (a) the canister moves $0.409c$ faster than the ship relative to the Earth, whereas in part (b) it moves $0.900c$ slower than the ship.

Doppler Shift

Although the speed of light does not change with relative velocity, the frequencies and wavelengths of light do. First discussed for sound waves, a Doppler shift occurs in any wave when there is relative motion between source and observer.

Relativistic Doppler Effects

The observed wavelength of electromagnetic radiation is longer (called a red shift) than that emitted by the source when the source moves away from the observer and shorter (called a blue shift) when the source moves towards the observer.

$$=\lambda_{\text{obs}} = \lambda_s \sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}. \tag{28.36}$$

In the Doppler equation, λ_{obs} is the observed wavelength, λ_s is the source wavelength, and u is the relative velocity of the source to the observer. The velocity u is positive for motion away from an observer and negative for motion toward an observer. In terms of source frequency and observed frequency, this equation can be written

$$f_{\text{obs}} = f_s \sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}}. \tag{28.37}$$

Notice that the – and + signs are different than in the wavelength equation.

Career Connection: Astronomer

If you are interested in a career that requires a knowledge of special relativity, there's probably no better connection than astronomy. Astronomers must take into account relativistic effects when they calculate distances, times, and speeds of black holes, galaxies, quasars, and all other astronomical objects. To have a career in astronomy, you need at least an undergraduate degree in either physics or astronomy, but a Master's or doctoral degree is often required. You also need a good background in high-level mathematics.

Example 28.5 Calculating a Doppler Shift: Radio Waves from a Receding Galaxy

Suppose a galaxy is moving away from the Earth at a speed $0.825c$. It emits radio waves with a wavelength of 0.525 m . What wavelength would we detect on the Earth?

Strategy

Because the galaxy is moving at a relativistic speed, we must determine the Doppler shift of the radio waves using the relativistic Doppler shift instead of the classical Doppler shift.

Solution

1. Identify the knowns. u=$0.825c$; $\lambda_s = 0.525\ m$
2. Identify the unknown. λ_{obs}
3. Choose the appropriate equation. $\lambda_{\text{obs}} = \lambda_s\sqrt{\dfrac{1+\frac{u}{c}}{1-\frac{u}{c}}}$
4. Plug the knowns into the equation.

$$\begin{aligned}\lambda_{\text{obs}} &= \lambda_s\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}\\ &= (0.525\text{ m})\sqrt{\frac{1+\frac{0.825c}{c}}{1-\frac{0.825c}{c}}}\\ &= 1.70\text{ m.}\end{aligned} \tag{28.38}$$

Discussion

Because the galaxy is moving away from the Earth, we expect the wavelengths of radiation it emits to be redshifted. The wavelength we calculated is 1.70 m, which is redshifted from the original wavelength of 0.525 m.

The relativistic Doppler shift is easy to observe. This equation has everyday applications ranging from Doppler-shifted radar velocity measurements of transportation to Doppler-radar storm monitoring. In astronomical observations, the relativistic Doppler shift provides velocity information such as the motion and distance of stars.

Check Your Understanding

Suppose a space probe moves away from the Earth at a speed $0.350c$. It sends a radio wave message back to the Earth at a frequency of 1.50 GHz. At what frequency is the message received on the Earth?

Solution

$$f_{\text{obs}} = f_s\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}} = (1.50\text{ GHz})\sqrt{\frac{1-\frac{0.350c}{c}}{1+\frac{0.350c}{c}}} = 1.04\text{ GHz} \tag{28.39}$$

28.5 Relativistic Momentum

Figure 28.18 Momentum is an important concept for these football players from the University of California at Berkeley and the University of California at Davis. Players with more mass often have a larger impact because their momentum is larger. For objects moving at relativistic speeds, the effect is even greater. (credit: John Martinez Pavliga)

In classical physics, momentum is a simple product of mass and velocity. However, we saw in the last section that when special relativity is taken into account, massive objects have a speed limit. What effect do you think mass and velocity have on the momentum of objects moving at relativistic speeds?

Momentum is one of the most important concepts in physics. The broadest form of Newton's second law is stated in terms of momentum. Momentum is conserved whenever the net external force on a system is zero. This makes momentum conservation a fundamental tool for analyzing collisions. All of **Work, Energy, and Energy Resources** is devoted to momentum, and momentum has been important for many other topics as well, particularly where collisions were involved. We will see that momentum has the same importance in modern physics. Relativistic momentum is conserved, and much of what we know about subatomic structure comes from the analysis of collisions of accelerator-produced relativistic particles.

The first postulate of relativity states that the laws of physics are the same in all inertial frames. Does the law of conservation of momentum survive this requirement at high velocities? The answer is yes, provided that the momentum is defined as follows.

Relativistic Momentum

Relativistic momentum p is classical momentum multiplied by the relativistic factor γ .

$$p = \gamma mu, \tag{28.40}$$

where m is the **rest mass** of the object, u is its velocity relative to an observer, and the relativistic factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}. \tag{28.41}$$

Note that we use u for velocity here to distinguish it from relative velocity v between observers. Only one observer is being considered here. With p defined in this way, total momentum p_{tot} is conserved whenever the net external force is zero, just as in classical physics. Again we see that the relativistic quantity becomes virtually the same as the classical at low velocities. That is, relativistic momentum γmu becomes the classical mu at low velocities, because γ is very nearly equal to 1 at low velocities.

Relativistic momentum has the same intuitive feel as classical momentum. It is greatest for large masses moving at high velocities, but, because of the factor γ , relativistic momentum approaches infinity as u approaches c . (See **Figure 28.19**.) This is another indication that an object with mass cannot reach the speed of light. If it did, its momentum would become infinite, an unreasonable value.

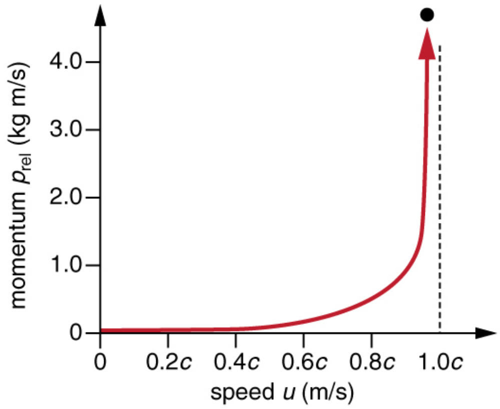


Figure 28.19 Relativistic momentum approaches infinity as the velocity of an object approaches the speed of light.

Misconception Alert: Relativistic Mass and Momentum

The relativistically correct definition of momentum as $p = \gamma mu$ is sometimes taken to imply that mass varies with velocity: $m_{\text{var}} = \gamma m$, particularly in older textbooks. However, note that m is the mass of the object as measured by a person at rest relative to the object. Thus, m is defined to be the rest mass, which could be measured at rest, perhaps using gravity. When a mass is moving relative to an observer, the only way that its mass can be determined is through collisions or other means in which momentum is involved. Since the mass of a moving object cannot be determined independently of momentum, the only meaningful mass is rest mass. Thus, when we use the term mass, assume it to be identical to rest mass.

Relativistic momentum is defined in such a way that the conservation of momentum will hold in all inertial frames. Whenever the net external force on a system is zero, relativistic momentum is conserved, just as is the case for classical momentum. This has been verified in numerous experiments.

In **Relativistic Energy**, the relationship of relativistic momentum to energy is explored. That subject will produce our first inkling that objects without mass may also have momentum.

Check Your Understanding

What is the momentum of an electron traveling at a speed $0.985c$? The rest mass of the electron is $9.11 \times 10^{-31}\ \text{kg}$.

Solution

$$p = \gamma mu = \frac{mu}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{(9.11 \times 10^{-31}\ \text{kg})(0.985)(3.00 \times 10^8\ \text{m/s})}{\sqrt{1 - \frac{(0.985c)^2}{c^2}}} = 1.56 \times 10^{-21}\ \text{kg} \cdot \text{m/s} \tag{28.42}$$

28.6 Relativistic Energy

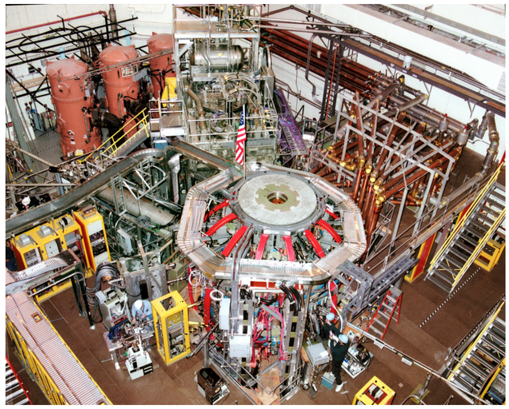


Figure 28.20 The National Spherical Torus Experiment (NSTX) has a fusion reactor in which hydrogen isotopes undergo fusion to produce helium. In this process, a relatively small mass of fuel is converted into a large amount of energy. (credit: Princeton Plasma Physics Laboratory)

A tokamak is a form of experimental fusion reactor, which can change mass to energy. Accomplishing this requires an understanding of relativistic energy. Nuclear reactors are proof of the conservation of relativistic energy.

Conservation of energy is one of the most important laws in physics. Not only does energy have many important forms, but each form can be converted to any other. We know that classically the total amount of energy in a system remains constant. Relativistically, energy is still conserved, provided its definition is altered to include the possibility of mass changing to energy, as in the reactions that occur within a nuclear reactor. Relativistic energy is intentionally defined so that it will be conserved in all inertial frames, just as is the case for relativistic momentum. As a consequence, we learn that several fundamental quantities are related in ways not known in classical physics. All of these relationships are verified by experiment and have fundamental consequences. The altered definition of energy contains some of the most fundamental and spectacular new insights into nature found in recent history.

Total Energy and Rest Energy

The first postulate of relativity states that the laws of physics are the same in all inertial frames. Einstein showed that the law of conservation of energy is valid relativistically, if we define energy to include a relativistic factor.

Total Energy

Total energy E is defined to be

$$E = \gamma mc^2, \tag{28.43}$$

where m is mass, c is the speed of light, $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, and v is the velocity of the mass relative to an observer. There are many aspects of the total energy E that we will discuss—among them are how kinetic and potential energies are included in E, and how E is related to relativistic momentum. But first, note that at rest, total energy is not zero. Rather, when $v = 0$, we have $\gamma = 1$, and an object has rest energy.

Rest Energy

Rest energy is

$$E_0 = mc^2. \tag{28.44}$$

This is the correct form of Einstein's most famous equation, which for the first time showed that energy is related to the mass of an object at rest. For example, if energy is stored in the object, its rest mass increases. This also implies that mass can be destroyed to release energy. The implications of these first two equations regarding relativistic energy are so broad that they were not completely recognized for some years after Einstein published them in 1907, nor was the experimental proof that they are correct widely recognized at first. Einstein, it should be noted, did understand and describe the meanings and implications of his theory.

Example 28.6 Calculating Rest Energy: Rest Energy is Very Large

Calculate the rest energy of a 1.00-g mass.

Strategy

One gram is a small mass—less than half the mass of a penny. We can multiply this mass, in SI units, by the speed of light squared to find the equivalent rest energy.

Solution

1. Identify the knowns. $m = 1.00 \times 10^{-3}\ \text{kg}$; $c = 3.00 \times 10^{8}\ \text{m/s}$

2. Identify the unknown. E_0
3. Choose the appropriate equation. $E_0 = mc^2$
4. Plug the knowns into the equation.

$$\begin{aligned} E_0 &= mc^2 = (1.00\times10^{-3}\text{ kg})(3.00\times10^{8}\text{ m/s})^2 \\ &= 9.00\times10^{13}\text{ kg}\cdot\text{m}^2/\text{s}^2 \end{aligned} \tag{28.45}$$

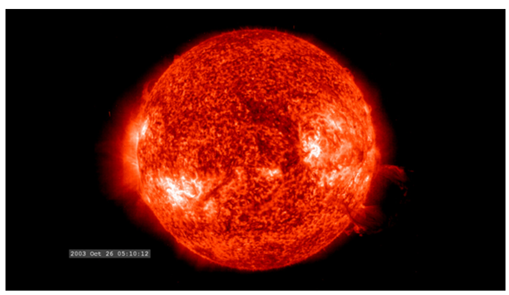
5. Convert units.
   Noting that $1\text{ kg}\cdot\text{m}^2/\text{s}^2 = 1\text{ J}$, we see the rest mass energy is

$$E_0 = 9.00\times10^{13}\text{ J}. \tag{28.46}$$

Discussion

This is an enormous amount of energy for a 1.00-g mass. We do not notice this energy, because it is generally not available. Rest energy is large because the speed of light c is a large number and c^2 is a very large number, so that mc^2 is huge for any macroscopic mass. The $9.00\times10^{13}\text{ J}$ rest mass energy for 1.00 g is about twice the energy released by the Hiroshima atomic bomb and about 10,000 times the kinetic energy of a large aircraft carrier. If a way can be found to convert rest mass energy into some other form (and all forms of energy can be converted into one another), then huge amounts of energy can be obtained from the destruction of mass.

Today, the practical applications of *the conversion of mass into another form of energy*, such as in nuclear weapons and nuclear power plants, are well known. But examples also existed when Einstein first proposed the correct form of relativistic energy, and he did describe some of them. Nuclear radiation had been discovered in the previous decade, and it had been a mystery as to where its energy originated. The explanation was that, in certain nuclear processes, a small amount of mass is destroyed and energy is released and carried by nuclear radiation. But the amount of mass destroyed is so small that it is difficult to detect that any is missing. Although Einstein proposed this as the source of energy in the radioactive salts then being studied, it was many years before there was broad recognition that mass could be and, in fact, commonly is converted to energy. (See **Figure 28.21**.)



(a)

(b)

Figure 28.21 The Sun (a) and the Susquehanna Steam Electric Station (b) both convert mass into energy—the Sun via nuclear fusion, the electric station via nuclear fission. (credits: (a) NASA/Goddard Space Flight Center, Scientific Visualization Studio; (b) U.S. government)

Because of the relationship of rest energy to mass, we now consider mass to be a form of energy rather than something separate. There had not even been a hint of this prior to Einstein's work. Such conversion is now known to be the source of the Sun's energy, the energy of nuclear decay, and even the source of energy keeping Earth's interior hot.

Stored Energy and Potential Energy

What happens to energy stored in an object at rest, such as the energy put into a battery by charging it, or the energy stored in a toy gun's compressed spring? The energy input becomes part of the total energy of the object and, thus, increases its rest mass. All stored and potential energy becomes mass in a system. Why is it we don't ordinarily notice this? In fact, conservation of mass (meaning total mass is constant) was one of the great laws verified by 19th-century science. Why was it not noticed to be incorrect? The following example helps answer these questions.

Example 28.7 Calculating Rest Mass: A Small Mass Increase due to Energy Input

A car battery is rated to be able to move 600 ampere-hours (A·h) of charge at 12.0 V. (a) Calculate the increase in rest mass of such a battery when it is taken from being fully depleted to being fully charged. (b) What percent increase is this, given the battery's mass is 20.0 kg?

Strategy

In part (a), we first must find the energy stored in the battery, which equals what the battery can supply in the form of electrical potential energy. Since $\text{PE}_{\text{elec}} = qV$, we have to calculate the charge q in $600\ \text{A·h}$, which is the product of the current I and the time t. We then multiply the result by 12.0 V. We can then calculate the battery's increase in mass using $\Delta E = \text{PE}_{\text{elec}} = (\Delta m)c^2$. Part (b) is a simple ratio converted to a percentage.

Solution for (a)

1. Identify the knowns. $I \cdot t = 600\ \text{A} \cdot \text{h}$; $V = 12.0\ \text{V}$; $c = 3.00 \times 10^8\ \text{m/s}$
2. Identify the unknown. Δm
3. Choose the appropriate equation. $\text{PE}_{\text{elec}} = (\Delta m)c^2$
4. Rearrange the equation to solve for the unknown. $\Delta m = \dfrac{\text{PE}_{\text{elec}}}{c^2}$
5. Plug the knowns into the equation.

$$\begin{aligned}\Delta m &= \frac{\text{PE}_{\text{elec}}}{c^2} \\ &= \frac{qV}{c^2} \\ &= \frac{(It)V}{c^2} \\ &= \frac{(600\ \text{A} \cdot \text{h})(12.0\ \text{V})}{(3.00 \times 10^8)^2}.\end{aligned} \tag{28.47}$$

Write amperes A as coulombs per second (C/s), and convert hours to seconds.

$$\begin{aligned}\Delta m &= \frac{(600\ \text{C/s} \cdot \text{h}\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)(12.0\ \text{J/C})}{(3.00 \times 10^8\ \text{m/s})^2} \\ &= \frac{(2.16 \times 10^6\ \text{C})(12.0\ \text{J/C})}{(3.00 \times 10^8\ \text{m/s})^2}\end{aligned} \tag{28.48}$$

Using the conversion $1\ \text{kg} \cdot \text{m}^2/\text{s}^2 = 1\ \text{J}$, we can write the mass as

$$\Delta m = 2.88 \times 10^{-10}\ \text{kg}.$$

Solution for (b)

1. Identify the knowns. $\Delta m = 2.88 \times 10^{-10}\ \text{kg}$; $m = 20.0\ \text{kg}$
2. Identify the unknown. % change
3. Choose the appropriate equation. $\%\ \text{increase} = \frac{\Delta m}{m} \times 100\%$
4. Plug the knowns into the equation.

$$\begin{aligned}\%\ \text{increase} &= \frac{\Delta m}{m} \times 100\% \\ &= \frac{2.88 \times 10^{-10}\ \text{kg}}{20.0\ \text{kg}} \times 100\% \\ &= 1.44 \times 10^{-9}\ \%.\end{aligned} \tag{28.49}$$

Discussion

Both the actual increase in mass and the percent increase are very small, since energy is divided by c^2, a very large number. We would have to be able to measure the mass of the battery to a precision of a billionth of a percent, or 1 part in 10^{11}, to notice this increase. It is no wonder that the mass variation is not readily observed. In fact, this change in mass is so small that we may question how you could verify it is real. The answer is found in nuclear processes in which the percentage of mass destroyed is large enough to be measured. The mass of the fuel of a nuclear reactor, for example, is measurably smaller when its energy has been used. In that case, stored energy has been released (converted mostly to heat and electricity) and the rest mass has decreased. This is also the case when you use the energy stored in a battery, except that the stored energy is much greater in nuclear processes, making the change in mass measurable in practice as well as in theory.

Kinetic Energy and the Ultimate Speed Limit

Kinetic energy is energy of motion. Classically, kinetic energy has the familiar expression $\frac{1}{2}mv^2$. The relativistic expression for kinetic energy is obtained from the work-energy theorem. This theorem states that the net work on a system goes into kinetic energy. If our system starts from rest, then the work-energy theorem is

$$W_{\text{net}} = \text{KE}. \tag{28.50}$$

Relativistically, at rest we have rest energy $E_0 = mc^2$. The work increases this to the total energy $E = \gamma mc^2$. Thus,

$$W_{\text{net}} = E - E_0 = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2. \tag{28.51}$$

Relativistically, we have $W_{\text{net}} = \text{KE}_{\text{rel}}$.

Relativistic Kinetic Energy

Relativistic kinetic energy is

$$\text{KE}_{\text{rel}} = (\gamma - 1)mc^2. \tag{28.52}$$

When motionless, we have $v = 0$ and

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1, \tag{28.53}$$

so that $\text{KE}_{\text{rel}} = 0$ at rest, as expected. But the expression for relativistic kinetic energy (such as total energy and rest energy) does not look much like the classical $\frac{1}{2}mv^2$. To show that the classical expression for kinetic energy is obtained at low velocities, we note that the binomial expansion for γ at low velocities gives

$$\gamma = 1 + \frac{1}{2}\frac{v^2}{c^2}. \tag{28.54}$$

A binomial expansion is a way of expressing an algebraic quantity as a sum of an infinite series of terms. In some cases, as in the limit of small velocity here, most terms are very small. Thus the expression derived for γ here is not exact, but it is a very accurate approximation. Thus, at low velocities,

$$\gamma - 1 = \frac{1}{2}\frac{v^2}{c^2}. \tag{28.55}$$

Entering this into the expression for relativistic kinetic energy gives

$$\text{KE}_{\text{rel}} = \left[\frac{1}{2}\frac{v^2}{c^2}\right]mc^2 = \frac{1}{2}mv^2 = \text{KE}_{\text{class}}. \tag{28.56}$$

So, in fact, relativistic kinetic energy does become the same as classical kinetic energy when $v << c$.

It is even more interesting to investigate what happens to kinetic energy when the velocity of an object approaches the speed of light. We know that γ becomes infinite as v approaches c , so that KE_{rel} also becomes infinite as the velocity approaches the speed of light. (See **Figure 28.22**.) An infinite amount of work (and, hence, an infinite amount of energy input) is required to accelerate a mass to the speed of light.

The Speed of Light

No object with mass can attain the speed of light.

So the speed of light is the ultimate speed limit for any particle having mass. All of this is consistent with the fact that velocities less than c always add to less than c . Both the relativistic form for kinetic energy and the ultimate speed limit being c have been confirmed in detail in numerous experiments. No matter how much energy is put into accelerating a mass, its velocity can only approach—not reach—the speed of light.

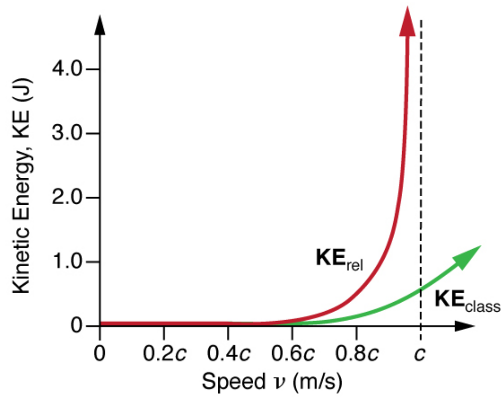


Figure 28.22 This graph of $\mathrm{KE}_{\mathrm{rel}}$ versus velocity shows how kinetic energy approaches infinity as velocity approaches the speed of light. It is thus not possible for an object having mass to reach the speed of light. Also shown is $\mathrm{KE}_{\mathrm{class}}$, the classical kinetic energy, which is similar to relativistic kinetic energy at low velocities. Note that much more energy is required to reach high velocities than predicted classically.

Example 28.8 Comparing Kinetic Energy: Relativistic Energy Versus Classical Kinetic Energy

An electron has a velocity $v = 0.990c$. (a) Calculate the kinetic energy in MeV of the electron. (b) Compare this with the classical value for kinetic energy at this velocity. (The mass of an electron is $9.11 \times 10^{-31}\ \mathrm{kg}$.)

Strategy

The expression for relativistic kinetic energy is always correct, but for (a) it must be used since the velocity is highly relativistic (close to c). First, we will calculate the relativistic factor γ, and then use it to determine the relativistic kinetic energy. For (b), we will calculate the classical kinetic energy (which would be close to the relativistic value if v were less than a few percent of c) and see that it is not the same.

Solution for (a)

1. Identify the knowns. $v = 0.990c$; $m = 9.11 \times 10^{-31}\ \mathrm{kg}$
2. Identify the unknown. $\mathrm{KE}_{\mathrm{rel}}$
3. Choose the appropriate equation. $\mathrm{KE}_{\mathrm{rel}} = (\gamma - 1)mc^2$
4. Plug the knowns into the equation.
   First calculate γ. We will carry extra digits because this is an intermediate calculation.

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \\ &= \frac{1}{\sqrt{1 - \frac{(0.990c)^2}{c^2}}} \\ &= \frac{1}{\sqrt{1 - (0.990)^2}} \\ &= 7.0888 \end{aligned} \tag{28.57}$$

   Next, we use this value to calculate the kinetic energy.

$$\begin{aligned} \mathrm{KE}_{\mathrm{rel}} &= (\gamma - 1)mc^2 \\ &= (7.0888 - 1)(9.11 \times 10^{-31}\ \mathrm{kg})(3.00 \times 10^{8}\ \mathrm{m/s})^2 \\ &= 4.99 \times 10^{-13}\ \mathrm{J} \end{aligned} \tag{28.58}$$

5. Convert units.

$$\begin{aligned} \mathrm{KE}_{\mathrm{rel}} &= (4.99 \times 10^{-13}\ \mathrm{J})\left(\frac{1\ \mathrm{MeV}}{1.60 \times 10^{-13}\ \mathrm{J}}\right) \\ &= 3.12\ \mathrm{MeV} \end{aligned} \tag{28.59}$$

Solution for (b)

1. List the knowns. $v = 0.990c$; $m = 9.11 \times 10^{-31}\ \mathrm{kg}$
2. List the unknown. $\mathrm{KE}_{\mathrm{class}}$

3. Choose the appropriate equation. $\mathrm{KE}_{\text{class}} = \frac{1}{2}mv^2$

4. Plug the knowns into the equation.

$$\begin{aligned}\mathrm{KE}_{\text{class}} &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(9.00\times10^{-31}\text{ kg})(0.990)^2(3.00\times10^{8}\text{ m/s})^2 \\ &= 4.02\times10^{-14}\text{ J}\end{aligned} \tag{28.60}$$

5. Convert units.

$$\begin{aligned}\mathrm{KE}_{\text{class}} &= 4.02\times10^{-14}\text{ J}\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) \\ &= 0.251\text{ MeV}\end{aligned} \tag{28.61}$$

Discussion

As might be expected, since the velocity is 99.0% of the speed of light, the classical kinetic energy is significantly off from the correct relativistic value. Note also that the classical value is much smaller than the relativistic value. In fact, $\mathrm{KE}_{\text{rel}}/\mathrm{KE}_{\text{class}} = 12.4$ here. This is some indication of how difficult it is to get a mass moving close to the speed of light. Much more energy is required than predicted classically. Some people interpret this extra energy as going into increasing the mass of the system, but, as discussed in **Relativistic Momentum**, this cannot be verified unambiguously. What is certain is that ever-increasing amounts of energy are needed to get the velocity of a mass a little closer to that of light. An energy of 3 MeV is a very small amount for an electron, and it can be achieved with present-day particle accelerators. SLAC, for example, can accelerate electrons to over $50\times10^9\text{ eV} = 50{,}000\text{ MeV}$.

Is there any point in getting v a little closer to c than 99.0% or 99.9%? The answer is yes. We learn a great deal by doing this. The energy that goes into a high-velocity mass can be converted to any other form, including into entirely new masses. (See **Figure 28.23**.) Most of what we know about the substructure of matter and the collection of exotic short-lived particles in nature has been learned this way. Particles are accelerated to extremely relativistic energies and made to collide with other particles, producing totally new species of particles. Patterns in the characteristics of these previously unknown particles hint at a basic substructure for all matter. These particles and some of their characteristics will be covered in **Particle Physics**.

Figure 28.23 The Fermi National Accelerator Laboratory, near Batavia, Illinois, was a subatomic particle collider that accelerated protons and antiprotons to attain energies up to 1 Tev (a trillion electronvolts). The circular ponds near the rings were built to dissipate waste heat. This accelerator was shut down in September 2011. (credit: Fermilab, Reidar Hahn)

Relativistic Energy and Momentum

We know classically that kinetic energy and momentum are related to each other, since

$$\mathrm{KE}_{\text{class}} = \frac{p^2}{2m} = \frac{(mv)^2}{2m} = \frac{1}{2}mv^2. \tag{28.62}$$

Relativistically, we can obtain a relationship between energy and momentum by algebraically manipulating their definitions. This produces

$$E^2 = (pc)^2 + (mc^2)^2, \tag{28.63}$$

where E is the relativistic total energy and p is the relativistic momentum. This relationship between relativistic energy and relativistic momentum is more complicated than the classical, but we can gain some interesting new insights by examining it. First, total energy is related to momentum and rest mass. At rest, momentum is zero, and the equation gives the total energy to be the rest energy mc^2 (so this equation is consistent with the discussion of rest energy above). However, as the mass is accelerated, its momentum p increases, thus increasing the total energy. At sufficiently high velocities, the rest energy term $(mc^2)^2$ becomes negligible compared with the momentum term $(pc)^2$; thus, $E = pc$ at extremely relativistic velocities.

If we consider momentum p to be distinct from mass, we can determine the implications of the equation $E^2 = (pc)^2 + (mc^2)^2,$ for a particle that has no mass. If we take m to be zero in this equation, then $E = pc$, or $p = E/c$. Massless particles have this momentum. There are several massless particles found in nature, including photons (these are quanta of electromagnetic radiation). Another implication is that a massless particle must travel at speed c and only at speed c. While it is beyond the scope of this text to examine the relationship in the equation $E^2 = (pc)^2 + (mc^2)^2,$ in detail, we can see that the relationship has important implications in special relativity.

Problem-Solving Strategies for Relativity

1. *Examine the situation to determine that it is necessary to use relativity*. Relativistic effects are related to $\gamma = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, the quantitative relativistic factor. If γ is very close to 1, then relativistic effects are small and differ very little from the usually easier classical calculations.
2. *Identify exactly what needs to be determined in the problem (identify the unknowns).*
3. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).* Look in particular for information on relative velocity v.
4. *Make certain you understand the conceptual aspects of the problem before making any calculations.* Decide, for example, which observer sees time dilated or length contracted before plugging into equations. If you have thought about who sees what, who is moving with the event being observed, who sees proper time, and so on, you will find it much easier to determine if your calculation is reasonable.
5. *Determine the primary type of calculation to be done to find the unknowns identified above.* You will find the section summary helpful in determining whether a length contraction, relativistic kinetic energy, or some other concept is involved.
6. *Do not round off during the calculation.* As noted in the text, you must often perform your calculations to many digits to see the desired effect. You may round off at the very end of the problem, but do not use a rounded number in a subsequent calculation.
7. *Check the answer to see if it is reasonable: Does it make sense?* This may be more difficult for relativity, since we do not encounter it directly. But you can look for velocities greater than c or relativistic effects that are in the wrong direction (such as a time contraction where a dilation was expected).

Check Your Understanding

A photon decays into an electron-positron pair. What is the kinetic energy of the electron if its speed is $0.992c$?

Solution

$$\begin{aligned}\text{KE}_{\text{rel}} &= (\gamma-1)mc^2 = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)mc^2 \\ &= \left(\frac{1}{\sqrt{1-\frac{(0.992c)^2}{c^2}}}-1\right)(9.11\times10^{-31}\text{ kg})(3.00\times10^{8}\text{ m/s})^2 = 5.67\times10^{-13}\text{ J}\end{aligned} \tag{28.64}$$

Glossary

classical velocity addition: the method of adding velocities when $v<<c$; velocities add like regular numbers in one-dimensional motion: $u = v+u'$, where v is the velocity between two observers, u is the velocity of an object relative to one observer, and u' is the velocity relative to the other observer

first postulate of special relativity: the idea that the laws of physics are the same and can be stated in their simplest form in all inertial frames of reference

inertial frame of reference: a reference frame in which a body at rest remains at rest and a body in motion moves at a constant speed in a straight line unless acted on by an outside force

length contraction: L, the shortening of the measured length of an object moving relative to the observer's frame: $L=L_0\sqrt{1-\frac{v^2}{c^2}} = \frac{L_0}{\gamma}$

Michelson-Morley experiment: an investigation performed in 1887 that proved that the speed of light in a vacuum is the same in all frames of reference from which it is viewed

proper length: L_0; the distance between two points measured by an observer who is at rest relative to both of the points; Earth-bound observers measure proper length when measuring the distance between two points that are stationary relative to the Earth

proper time: Δt_0. the time measured by an observer at rest relative to the event being observed: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\Delta t_0$, where $\gamma = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

relativistic Doppler effects: a change in wavelength of radiation that is moving relative to the observer; the wavelength of the radiation is longer (called a red shift) than that emitted by the source when the source moves away from the observer and shorter (called a blue shift) when the source moves toward the observer; the shifted wavelength is described by the equation

$$\lambda_{\text{obs}} = \lambda_s \sqrt{\frac{1 + \frac{u}{c}}{1 - \frac{u}{c}}}$$

where λ_{obs} is the observed wavelength, λ_s is the source wavelength, and u is the velocity of the source to the observer

relativistic kinetic energy: the kinetic energy of an object moving at relativistic speeds: $\text{KE}_{\text{rel}} = (\gamma - 1)mc^2$, where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

relativistic momentum: p , the momentum of an object moving at relativistic velocity; $p = \gamma mu$, where m is the rest mass of the object, u is its velocity relative to an observer, and the relativistic factor $\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$

relativistic velocity addition: the method of adding velocities of an object moving at a relativistic speed: $u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$, where v is the relative velocity between two observers, u is the velocity of an object relative to one observer, and u' is the velocity relative to the other observer

relativity: the study of how different observers measure the same event

rest energy: the energy stored in an object at rest: $E_0 = mc^2$

rest mass: the mass of an object as measured by a person at rest relative to the object

second postulate of special relativity: the idea that the speed of light c is a constant, independent of the source

special relativity: the theory that, in an inertial frame of reference, the motion of an object is relative to the frame from which it is viewed or measured

time dilation: the phenomenon of time passing slower to an observer who is moving relative to another observer

total energy: defined as $E = \gamma mc^2$, where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

twin paradox: this asks why a twin traveling at a relativistic speed away and then back towards the Earth ages less than the Earth-bound twin. The premise to the paradox is faulty because the traveling twin is accelerating, and special relativity does not apply to accelerating frames of reference

Section Summary

28.1 Einstein's Postulates

- Relativity is the study of how different observers measure the same event.
- Modern relativity is divided into two parts. Special relativity deals with observers who are in uniform (unaccelerated) motion, whereas general relativity includes accelerated relative motion and gravity. Modern relativity is correct in all circumstances and, in the limit of low velocity and weak gravitation, gives the same predictions as classical relativity.
- An inertial frame of reference is a reference frame in which a body at rest remains at rest and a body in motion moves at a constant speed in a straight line unless acted on by an outside force.
- Modern relativity is based on Einstein's two postulates. The first postulate of special relativity is the idea that the laws of physics are the same and can be stated in their simplest form in all inertial frames of reference. The second postulate of special relativity is the idea that the speed of light c is a constant, independent of the relative motion of the source.
- The Michelson-Morley experiment demonstrated that the speed of light in a vacuum is independent of the motion of the Earth about the Sun.

28.2 Simultaneity And Time Dilation

- Two events are defined to be simultaneous if an observer measures them as occurring at the same time. They are not necessarily simultaneous to all observers—simultaneity is not absolute.
- Time dilation is the phenomenon of time passing slower for an observer who is moving relative to another observer.
- Observers moving at a relative velocity v do not measure the same elapsed time for an event. Proper time Δt_0 is the time measured by an observer at rest relative to the event being observed. Proper time is related to the time Δt measured by an Earth-bound observer by the equation

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \Delta t_0,$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

- The equation relating proper time and time measured by an Earth-bound observer implies that relative velocity cannot exceed the speed of light.
- The twin paradox asks why a twin traveling at a relativistic speed away and then back towards the Earth ages less than the Earth-bound twin. The premise to the paradox is faulty because the traveling twin is accelerating. Special relativity does not apply to accelerating frames of reference.
- Time dilation is usually negligible at low relative velocities, but it does occur, and it has been verified by experiment.

28.3 Length Contraction

- All observers agree upon relative speed.
- Distance depends on an observer's motion. Proper length L_0 is the distance between two points measured by an observer who is at rest relative to both of the points. Earth-bound observers measure proper length when measuring the distance between two points that are stationary relative to the Earth.
- Length contraction L is the shortening of the measured length of an object moving relative to the observer's frame:

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}} = \frac{L_0}{\gamma}.$$

28.4 Relativistic Addition of Velocities

- With classical velocity addition, velocities add like regular numbers in one-dimensional motion: $u = v + u'$, where v is the velocity between two observers, u is the velocity of an object relative to one observer, and u' is the velocity relative to the other observer.
- Velocities cannot add to be greater than the speed of light. Relativistic velocity addition describes the velocities of an object moving at a relativistic speed:

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$

- An observer of electromagnetic radiation sees **relativistic Doppler effects** if the source of the radiation is moving relative to the observer. The wavelength of the radiation is longer (called a red shift) than that emitted by the source when the source moves away from the observer and shorter (called a blue shift) when the source moves toward the observer. The shifted wavelength is described by the equation

$$\lambda_{\text{obs}} = \lambda_s\sqrt{\frac{1 + \frac{u}{c}}{1 - \frac{u}{c}}}$$

λ_{obs} is the observed wavelength, λ_s is the source wavelength, and u is the relative velocity of the source to the observer.

28.5 Relativistic Momentum

- The law of conservation of momentum is valid whenever the net external force is zero and for relativistic momentum. Relativistic momentum p is classical momentum multiplied by the relativistic factor γ.
- $p = \gamma mu$, where m is the rest mass of the object, u is its velocity relative to an observer, and the relativistic factor $\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$.
- At low velocities, relativistic momentum is equivalent to classical momentum.
- Relativistic momentum approaches infinity as u approaches c. This implies that an object with mass cannot reach the speed of light.
- Relativistic momentum is conserved, just as classical momentum is conserved.

28.6 Relativistic Energy

- Relativistic energy is conserved as long as we define it to include the possibility of mass changing to energy.
- Total Energy is defined as: $E = \gamma mc^2$, where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$.
- Rest energy is $E_0 = mc^2$, meaning that mass is a form of energy. If energy is stored in an object, its mass increases. Mass can be destroyed to release energy.

- We do not ordinarily notice the increase or decrease in mass of an object because the change in mass is so small for a large increase in energy.
- The relativistic work-energy theorem is $W_{\text{net}} = E - E_0 = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$.
- Relativistically, $W_{\text{net}} = \text{KE}_{\text{rel}}$, where KE_{rel} is the relativistic kinetic energy.
- Relativistic kinetic energy is $\text{KE}_{\text{rel}} = (\gamma - 1)mc^2$, where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$. At low velocities, relativistic kinetic energy reduces to classical kinetic energy.
- **No object with mass can attain the speed of light** because an infinite amount of work and an infinite amount of energy input is required to accelerate a mass to the speed of light.
- The equation $E^2 = (pc)^2 + (mc^2)^2$ relates the relativistic total energy E and the relativistic momentum p. At extremely high velocities, the rest energy mc^2 becomes negligible, and $E = pc$.

Conceptual Questions

28.1 Einstein's Postulates

1. Which of Einstein's postulates of special relativity includes a concept that does not fit with the ideas of classical physics? Explain.

2. Is Earth an inertial frame of reference? Is the Sun? Justify your response.

3. When you are flying in a commercial jet, it may appear to you that the airplane is stationary and the Earth is moving beneath you. Is this point of view valid? Discuss briefly.

28.2 Simultaneity And Time Dilation

4. Does motion affect the rate of a clock as measured by an observer moving with it? Does motion affect how an observer moving relative to a clock measures its rate?

5. To whom does the elapsed time for a process seem to be longer, an observer moving relative to the process or an observer moving with the process? Which observer measures proper time?

6. How could you travel far into the future without aging significantly? Could this method also allow you to travel into the past?

28.3 Length Contraction

7. To whom does an object seem greater in length, an observer moving with the object or an observer moving relative to the object? Which observer measures the object's proper length?

8. Relativistic effects such as time dilation and length contraction are present for cars and airplanes. Why do these effects seem strange to us?

9. Suppose an astronaut is moving relative to the Earth at a significant fraction of the speed of light. (a) Does he observe the rate of his clocks to have slowed? (b) What change in the rate of Earth-bound clocks does he see? (c) Does his ship seem to him to shorten? (d) What about the distance between stars that lie on lines parallel to his motion? (e) Do he and an Earth-bound observer agree on his velocity relative to the Earth?

28.4 Relativistic Addition of Velocities

10. Explain the meaning of the terms "red shift" and "blue shift" as they relate to the relativistic Doppler effect.

11. What happens to the relativistic Doppler effect when relative velocity is zero? Is this the expected result?

12. Is the relativistic Doppler effect consistent with the classical Doppler effect in the respect that λ_{obs} is larger for motion away?

13. All galaxies farther away than about $50 \times 10^6\ \text{ly}$ exhibit a red shift in their emitted light that is proportional to distance, with those farther and farther away having progressively greater red shifts. What does this imply, assuming that the only source of red shift is relative motion? (Hint: At these large distances, it is space itself that is expanding, but the effect on light is the same.)

28.5 Relativistic Momentum

14. How does modern relativity modify the law of conservation of momentum?

15. Is it possible for an external force to be acting on a system and relativistic momentum to be conserved? Explain.

28.6 Relativistic Energy

16. How are the classical laws of conservation of energy and conservation of mass modified by modern relativity?

17. What happens to the mass of water in a pot when it cools, assuming no molecules escape or are added? Is this observable in practice? Explain.

18. Consider a thought experiment. You place an expanded balloon of air on weighing scales outside in the early morning. The balloon stays on the scales and you are able to measure changes in its mass. Does the mass of the balloon change as the day progresses? Discuss the difficulties in carrying out this experiment.

19. The mass of the fuel in a nuclear reactor decreases by an observable amount as it puts out energy. Is the same true for the coal and oxygen combined in a conventional power plant? If so, is this observable in practice for the coal and oxygen? Explain.

20. We know that the velocity of an object with mass has an upper limit of c. Is there an upper limit on its momentum? Its energy? Explain.

21. Given the fact that light travels at c, can it have mass? Explain.

22. If you use an Earth-based telescope to project a laser beam onto the Moon, you can move the spot across the Moon's surface at a velocity greater than the speed of light. Does this violate modern relativity? (Note that light is being sent from the Earth to the Moon, not across the surface of the Moon.)

Problems & Exercises

28.2 Simultaneity And Time Dilation

23. (a) What is γ if $v = 0.250c$? (b) If $v = 0.500c$?

24. (a) What is γ if $v = 0.100c$? (b) If $v = 0.900c$?

25. Particles called π -mesons are produced by accelerator beams. If these particles travel at 2.70×10^8 m/s and live 2.60×10^{-8} s when at rest relative to an observer, how long do they live as viewed in the laboratory?

26. Suppose a particle called a kaon is created by cosmic radiation striking the atmosphere. It moves by you at $0.980c$, and it lives 1.24×10^{-8} s when at rest relative to an observer. How long does it live as you observe it?

27. A neutral π -meson is a particle that can be created by accelerator beams. If one such particle lives 1.40×10^{-16} s as measured in the laboratory, and 0.840×10^{-16} s when at rest relative to an observer, what is its velocity relative to the laboratory?

28. A neutron lives 900 s when at rest relative to an observer. How fast is the neutron moving relative to an observer who measures its life span to be 2065 s?

29. If relativistic effects are to be less than 1%, then γ must be less than 1.01. At what relative velocity is $\gamma = 1.01$?

30. If relativistic effects are to be less than 3%, then γ must be less than 1.03. At what relative velocity is $\gamma = 1.03$?

31. (a) At what relative velocity is $\gamma = 1.50$? (b) At what relative velocity is $\gamma = 100$?

32. (a) At what relative velocity is $\gamma = 2.00$? (b) At what relative velocity is $\gamma = 10.0$?

33. Unreasonable Results

(a) Find the value of γ for the following situation. An Earth-bound observer measures 23.9 h to have passed while signals from a high-velocity space probe indicate that 24.0 h have passed on board. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

28.3 Length Contraction

34. A spaceship, 200 m long as seen on board, moves by the Earth at $0.970c$. What is its length as measured by an Earth-bound observer?

35. How fast would a 6.0 m-long sports car have to be going past you in order for it to appear only 5.5 m long?

36. (a) How far does the muon in **Example 28.1** travel according to the Earth-bound observer? (b) How far does it travel as viewed by an observer moving with it? Base your calculation on its velocity relative to the Earth and the time it lives (proper time). (c) Verify that these two distances are related through length contraction $\gamma = 3.20$.

37. (a) How long would the muon in **Example 28.1** have lived as observed on the Earth if its velocity was $0.0500c$? (b) How far would it have traveled as observed on the Earth? (c) What distance is this in the muon's frame?

38. (a) How long does it take the astronaut in **Example 28.2** to travel 4.30 ly at $0.99944c$ (as measured by the Earth-bound observer)? (b) How long does it take according to the astronaut? (c) Verify that these two times are related through time dilation with $\gamma = 30.00$ as given.

39. (a) How fast would an athlete need to be running for a 100-m race to look 100 yd long? (b) Is the answer consistent with the fact that relativistic effects are difficult to observe in ordinary circumstances? Explain.

40. Unreasonable Results

(a) Find the value of γ for the following situation. An astronaut measures the length of her spaceship to be 25.0 m, while an Earth-bound observer measures it to be 100 m. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

41. Unreasonable Results

A spaceship is heading directly toward the Earth at a velocity of $0.800c$. The astronaut on board claims that he can send a canister toward the Earth at $1.20c$ relative to the Earth. (a) Calculate the velocity the canister must have relative to the spaceship. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

28.4 Relativistic Addition of Velocities

42. Suppose a spaceship heading straight towards the Earth at $0.750c$ can shoot a canister at $0.500c$ relative to the ship. (a) What is the velocity of the canister relative to the Earth, if it is shot directly at the Earth? (b) If it is shot directly away from the Earth?

43. Repeat the previous problem with the ship heading directly away from the Earth.

44. If a spaceship is approaching the Earth at $0.100c$ and a message capsule is sent toward it at $0.100c$ relative to the Earth, what is the speed of the capsule relative to the ship?

45. (a) Suppose the speed of light were only 3000 m/s . A jet fighter moving toward a target on the ground at 800 m/s shoots bullets, each having a muzzle velocity of 1000 m/s . What are the bullets' velocity relative to the target? (b) If the speed of light was this small, would you observe relativistic effects in everyday life? Discuss.

46. If a galaxy moving away from the Earth has a speed of 1000 km/s and emits 656 nm light characteristic of hydrogen (the most common element in the universe). (a) What wavelength would we observe on the Earth? (b) What type of electromagnetic radiation is this? (c) Why is the speed of the Earth in its orbit negligible here?

47. A space probe speeding towards the nearest star moves at $0.250c$ and sends radio information at a broadcast frequency of 1.00 GHz. What frequency is received on the Earth?

48. If two spaceships are heading directly towards each other at $0.800c$, at what speed must a canister be shot from the first ship to approach the other at $0.999c$ as seen by the second ship?

49. Two planets are on a collision course, heading directly towards each other at $0.250c$. A spaceship sent from one planet approaches the second at $0.750c$ as seen by the second planet. What is the velocity of the ship relative to the first planet?

50. When a missile is shot from one spaceship towards another, it leaves the first at $0.950c$ and approaches the other at $0.750c$. What is the relative velocity of the two ships?

51. What is the relative velocity of two spaceships if one fires a missile at the other at $0.750c$ and the other observes it to approach at $0.950c$?

52. Near the center of our galaxy, hydrogen gas is moving directly away from us in its orbit about a black hole. We receive 1900 nm electromagnetic radiation and know that it was 1875 nm when emitted by the hydrogen gas. What is the speed of the gas?

53. A highway patrol officer uses a device that measures the speed of vehicles by bouncing radar off them and measuring the Doppler shift. The outgoing radar has a frequency of 100 GHz and the returning echo has a frequency 15.0 kHz higher. What is the velocity of the vehicle? Note that there are two Doppler shifts in echoes. Be certain not to round off until the end of the problem, because the effect is small.

54. Prove that for any relative velocity v between two observers, a beam of light sent from one to the other will approach at speed c (provided that v is less than c, of course).

55. Show that for any relative velocity v between two observers, a beam of light projected by one directly away from the other will move away at the speed of light (provided that v is less than c, of course).

56. (a) All but the closest galaxies are receding from our own Milky Way Galaxy. If a galaxy $12.0\times10^9\ \text{ly}$ ly away is receding from us at 0. $0.900c$, at what velocity relative to us must we send an exploratory probe to approach the other galaxy at $0.990c$, as measured from that galaxy? (b) How long will it take the probe to reach the other galaxy as measured from the Earth? You may assume that the velocity of the other galaxy remains constant. (c) How long will it then take for a radio signal to be beamed back? (All of this is possible in principle, but not practical.)

28.5 Relativistic Momentum

57. Find the momentum of a helium nucleus having a mass of $6.68\times10^{-27}\ \text{kg}$ that is moving at $0.200c$.

58. What is the momentum of an electron traveling at $0.980c$?

59. (a) Find the momentum of a $1.00\times10^9\ \text{kg}$ asteroid heading towards the Earth at $30.0\ \text{km/s}$. (b) Find the ratio of this momentum to the classical momentum. (Hint: Use the approximation that $\gamma = 1 + (1/2)v^2/c^2$ at low velocities.)

60. (a) What is the momentum of a 2000 kg satellite orbiting at 4.00 km/s? (b) Find the ratio of this momentum to the classical momentum. (Hint: Use the approximation that $\gamma = 1 + (1/2)v^2/c^2$ at low velocities.)

61. What is the velocity of an electron that has a momentum of $3.04\times10^{-21}\ \text{kg}\cdot\text{m/s}$? Note that you must calculate the velocity to at least four digits to see the difference from c.

62. Find the velocity of a proton that has a momentum of $4.48\times\text{–}10^{\text{-}19}\ \text{kg}\cdot\text{m/s.}$

63. (a) Calculate the speed of a $1.00\text{-}\mu\text{g}$ particle of dust that has the same momentum as a proton moving at $0.999c$. (b) What does the small speed tell us about the mass of a proton compared to even a tiny amount of macroscopic matter?

64. (a) Calculate γ for a proton that has a momentum of $1.00\ \text{kg}\cdot\text{m/s.}$ (b) What is its speed? Such protons form a rare component of cosmic radiation with uncertain origins.

28.6 Relativistic Energy

65. What is the rest energy of an electron, given its mass is $9.11\times10^{-31}\ \text{kg}$? Give your answer in joules and MeV.

66. Find the rest energy in joules and MeV of a proton, given its mass is $1.67\times10^{-27}\ \text{kg}$.

67. If the rest energies of a proton and a neutron (the two constituents of nuclei) are 938.3 and 939.6 MeV respectively, what is the difference in their masses in kilograms?

68. The Big Bang that began the universe is estimated to have released $10^{68}\ \text{J}$ of energy. How many stars could half this energy create, assuming the average star's mass is $4.00\times10^{30}\ \text{kg}$?

69. A supernova explosion of a $2.00\times10^{31}\ \text{kg}$ star produces $1.00\times10^{44}\ \text{kg}$ of energy. (a) How many kilograms of mass are converted to energy in the explosion? (b) What is the ratio $\Delta m/m$ of mass destroyed to the original mass of the star?

70. (a) Using data from **Table 7.1**, calculate the mass converted to energy by the fission of 1.00 kg of uranium. (b) What is the ratio of mass destroyed to the original mass, $\Delta m/m$?

71. (a) Using data from **Table 7.1**, calculate the amount of mass converted to energy by the fusion of 1.00 kg of hydrogen. (b) What is the ratio of mass destroyed to the original mass, $\Delta m/m$? (c) How does this compare with $\Delta m/m$ for the fission of 1.00 kg of uranium?

72. There is approximately $10^{34}\ \text{J}$ of energy available from fusion of hydrogen in the world's oceans. (a) If $10^{33}\ \text{J}$ of this energy were utilized, what would be the decrease in mass of the oceans? (b) How great a volume of water does this correspond to? (c) Comment on whether this is a significant fraction of the total mass of the oceans.

73. A muon has a rest mass energy of 105.7 MeV, and it decays into an electron and a massless particle. (a) If all the lost mass is converted into the electron's kinetic energy, find γ for the electron. (b) What is the electron's velocity?

74. A π-meson is a particle that decays into a muon and a massless particle. The π-meson has a rest mass energy of 139.6 MeV, and the muon has a rest mass energy of 105.7 MeV. Suppose the π-meson is at rest and all of the missing mass goes into the muon's kinetic energy. How fast will the muon move?

75. (a) Calculate the relativistic kinetic energy of a 1000-kg car moving at 30.0 m/s if the speed of light were only 45.0 m/s. (b) Find the ratio of the relativistic kinetic energy to classical.

76. Alpha decay is nuclear decay in which a helium nucleus is emitted. If the helium nucleus has a mass of $6.80\times10^{-27}\ \text{kg}$ and is given 5.00 MeV of kinetic energy, what is its velocity?

77. (a) Beta decay is nuclear decay in which an electron is emitted. If the electron is given 0.750 MeV of kinetic energy, what is its velocity? (b) Comment on how the high velocity is consistent with the kinetic energy as it compares to the rest mass energy of the electron.

78. A positron is an antimatter version of the electron, having exactly the same mass. When a positron and an electron meet, they annihilate, converting all of their mass into energy. (a) Find the energy released, assuming negligible kinetic energy before the annihilation. (b) If this energy is given to a proton in the form of kinetic energy, what is its velocity? (c) If this energy is given to another electron in the form of kinetic energy, what is its velocity?

79. What is the kinetic energy in MeV of a π-meson that lives $1.40\times10^{-16}\ \text{s}$ as measured in the laboratory, and $0.840\times10^{-16}\ \text{s}$ when at rest relative to an observer, given that its rest energy is 135 MeV?

80. Find the kinetic energy in MeV of a neutron with a measured life span of 2065 s, given its rest energy is 939.6 MeV, and rest life span is 900s.

81. (a) Show that $(pc)^2/(mc^2)^2 = \gamma^2 - 1$. This means that at large velocities $pc >> mc^2$. (b) Is $E \approx pc$ when $\gamma = 30.0$, as for the astronaut discussed in the twin paradox?

82. One cosmic ray neutron has a velocity of $0.250c$ relative to the Earth. (a) What is the neutron's total energy in MeV? (b) Find its momentum. (c) Is $E \approx pc$ in this situation? Discuss in terms of the equation given in part (a) of the previous problem.

83. What is γ for a proton having a mass energy of 938.3 MeV accelerated through an effective potential of 1.0 TV (teravolt) at Fermilab outside Chicago?

84. (a) What is the effective accelerating potential for electrons at the Stanford Linear Accelerator, if $\gamma = 1.00\times10^5$ for them? (b) What is their total energy (nearly the same as kinetic in this case) in GeV?

85. (a) Using data from **Table 7.1**, find the mass destroyed when the energy in a barrel of crude oil is released. (b) Given these barrels contain 200 liters and assuming the density of crude oil is $750\ \text{kg/m}^3$, what is the ratio of mass destroyed to original mass, $\Delta m / m$?

86. (a) Calculate the energy released by the destruction of 1.00 kg of mass. (b) How many kilograms could be lifted to a 10.0 km height by this amount of energy?

87. A Van de Graaff accelerator utilizes a 50.0 MV potential difference to accelerate charged particles such as protons. (a) What is the velocity of a proton accelerated by such a potential? (b) An electron?

88. Suppose you use an average of $500\ \text{kW}\cdot\text{h}$ of electric energy per month in your home. (a) How long would 1.00 g of mass converted to electric energy with an efficiency of 38.0% last you? (b) How many homes could be supplied at the $500\ \text{kW}\cdot\text{h}$ per month rate for one year by the energy from the described mass conversion?

89. (a) A nuclear power plant converts energy from nuclear fission into electricity with an efficiency of 35.0%. How much mass is destroyed in one year to produce a continuous 1000 MW of electric power? (b) Do you think it would be possible to observe this mass loss if the total mass of the fuel is $10^4\ \text{kg}$?

90. Nuclear-powered rockets were researched for some years before safety concerns became paramount. (a) What fraction of a rocket's mass would have to be destroyed to get it into a low Earth orbit, neglecting the decrease in gravity? (Assume an orbital altitude of 250 km, and calculate both the kinetic energy (classical) and the gravitational potential energy needed.) (b) If the ship has a mass of $1.00\times10^5\ \text{kg}$ (100 tons), what total yield nuclear explosion in tons of TNT is needed?

91. The Sun produces energy at a rate of 4.00×10^{26} W by the fusion of hydrogen. (a) How many kilograms of hydrogen undergo fusion each second? (b) If the Sun is 90.0% hydrogen and half of this can undergo fusion before the Sun changes character, how long could it produce energy at its current rate? (c) How many kilograms of mass is the Sun losing per second? (d) What fraction of its mass will it have lost in the time found in part (b)?

92. Unreasonable Results

A proton has a mass of $1.67\times10^{-27}\ \text{kg}$. A physicist measures the proton's total energy to be 50.0 MeV. (a) What is the proton's kinetic energy? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

93. Construct Your Own Problem

Consider a highly relativistic particle. Discuss what is meant by the term "highly relativistic." (Note that, in part, it means that the particle cannot be massless.) Construct a problem in which you calculate the wavelength of such a particle and show that it is very nearly the same as the wavelength of a massless particle, such as a photon, with the same energy. Among the things to be considered are the rest energy of the particle (it should be a known particle) and its total energy, which should be large compared to its rest energy.

94. Construct Your Own Problem

Consider an astronaut traveling to another star at a relativistic velocity. Construct a problem in which you calculate the time for the trip as observed on the Earth and as observed by the astronaut. Also calculate the amount of mass that must be converted to energy to get the astronaut and ship to the velocity travelled. Among the things to be considered are the distance to the star, the velocity, and the mass of the astronaut and ship. Unless your instructor directs you otherwise, do not include any energy given to other masses, such as rocket propellants.